



1. Equivalent moment of Inertia & Torque  
Rotational Motion & Translational Motion.

Rotational Motion:

Load with rotational motion, It consist of motor, two load and gears, the motor driving two loads, one load is directly connected to shaft and other one through gear with  $n$  and  $n_1$ .

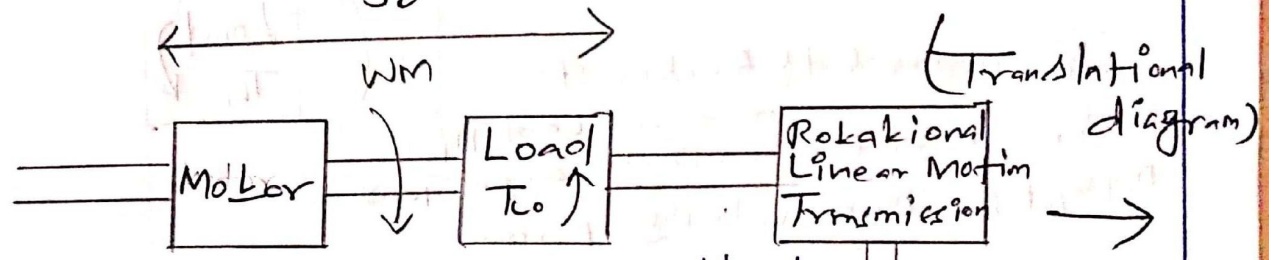
Gear teeth Ratio  $a_1 = \frac{n}{n_1} = \frac{\omega_{m1}}{\omega_m}$

Kinetic Energy is equal to kinetic energy various part

$$\frac{1}{2} J \omega_m^2 = \frac{1}{2} J_0 \omega_m^2 + \frac{1}{2} J_1 \omega_{m1}^2$$

$$J \omega_m^2 = J_0 \omega_m^2 + J_1 \omega_{m1}^2$$

$$\frac{J}{\omega_m^2} = \frac{J_0 \omega_m^2}{\omega_m^2} + \frac{J_1 \omega_{m1}^2}{\omega_m^2}$$



$J_0$  - moment of inertia of motor and load

$\omega_m$  - motor speed

$T_L$  - load torque

$J_1$  - moment of inertia of the load connected to gear

$\omega_{m1}$  - speed of the load coupled through gear

$T_{L1}$  - load torque (directly connected to gear)

$$J = J_0 + J_1 \frac{\omega_{m1}^2}{\omega_m^2} \quad a_1 = \frac{\omega_{m1}}{\omega_m}$$

$$J = J_0 + a_1^2 J_1$$

Power at the Motor =  $T_L \times \omega_m$

Power at the load =  $T_{L0} \times \omega_m$

Power at Load  $T_L = \frac{T_L \omega_m}{\eta_1}$

power at load & motor must be equal

$T_L \omega_m = T_L \omega_m + \frac{T_L \omega_m}{\eta_1}$   
 + equal in  $\omega_m$

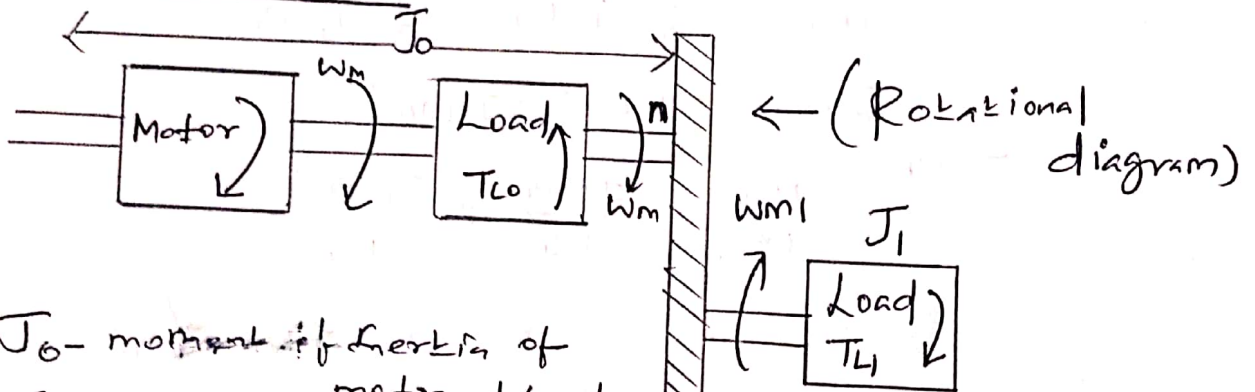
$$\frac{T_L \omega_m}{\omega_m} = \frac{T_L \omega_m}{\omega_m} + \frac{T_L \omega_m}{\eta_1 \omega_m}$$

$$T_L = T_L + \frac{T_L}{\eta_1} a_1$$

$$J = J_0 + a_1^2 J_1 + a_2^2 J_2 + \dots + a_n^2 J_n$$

$$T_L = T_L + \frac{a_1 T_L}{\eta_1} + \frac{a_2 T_L}{\eta_2} + \dots + \frac{a_n T_L}{\eta_n}$$

Translational Motion:-



$J_0$  - moment of inertia of motor and load  $n$   
 $T_{L0}$  - load torque directly coupled  
 $M_1, v_1, F_1$  - mass, velocity force

Thus  $\frac{1}{2} J \omega_m^2 = \frac{1}{2} J_0 \omega_m^2 + \frac{1}{2} M_1 v_1^2$

$$J \omega_m^2 = J_0 \omega_m^2 + M_1 v_1^2$$

$$\frac{J \omega_m^2}{\omega_m^2} = \frac{J_0 \omega_m^2}{\omega_m^2} + \frac{M_1 v_1^2}{\omega_m^2}$$

$$J = J_0 + M_1 \left( \frac{v_1}{\omega_m} \right)^2$$

Power at the Motor =  $T_L \omega_m$

power at the Load =  $T_{L0} \omega_m$

Power at the translation =  $\frac{F_1 v_1}{\eta_1}$

$$T_L \omega_m = T_{L0} \omega_m + \frac{F_1 v_1}{\eta_1}$$

+  $\omega_m$



$$\frac{T_L \omega_m}{\omega_m} = \frac{T_{L0} \omega_m}{\omega_m} + \frac{F_1 V_1}{\eta_1 \omega_m}$$

$$T_L = T_{L0} + \frac{F_1}{\eta_1} \left( \frac{V_1}{\omega_m} \right)$$

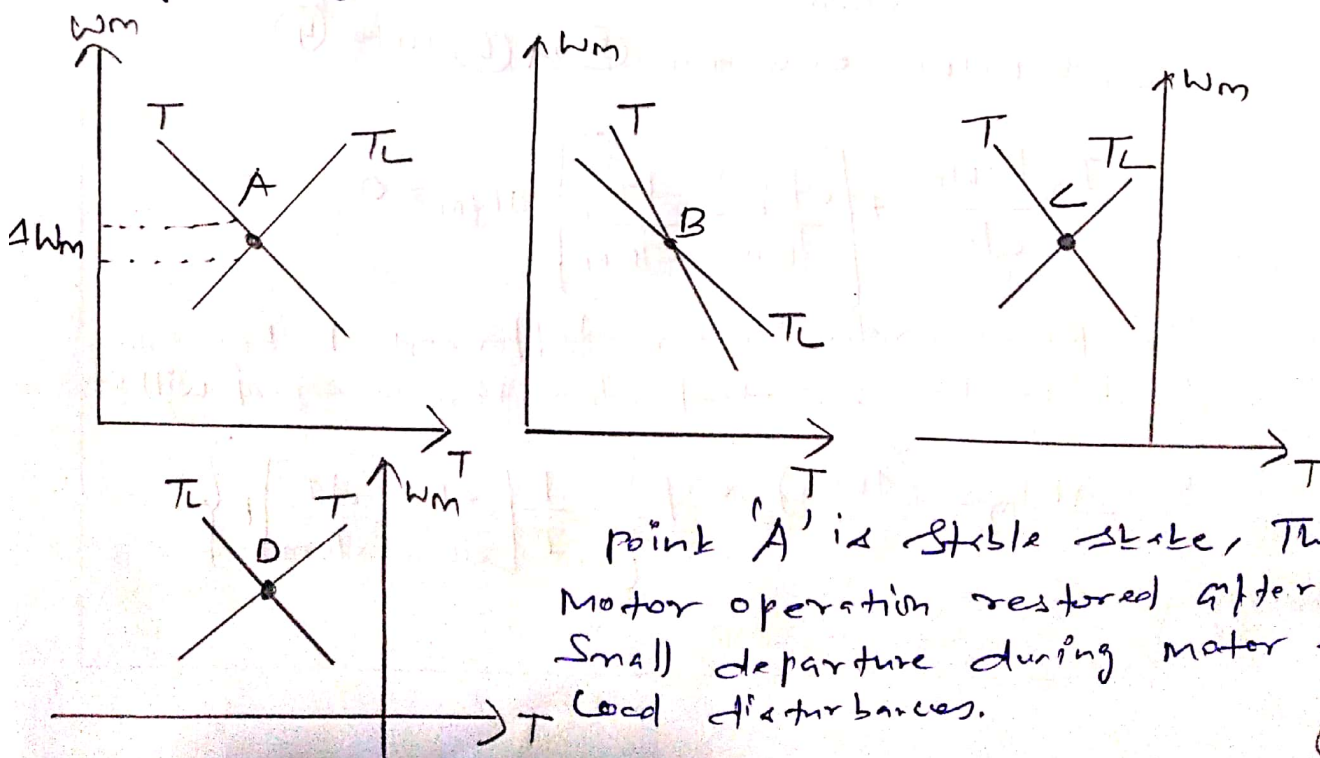
$$\therefore J = J_0 + M_1 \left( \frac{V_1}{\omega_m} \right)^2 + M_2 \left( \frac{V_2}{\omega_m} \right)^2 + \dots + M_n \left( \frac{V_n}{\omega_m} \right)^2$$

$$T_L = T_{L0} + \frac{F_1}{\eta_1} \left( \frac{V_1}{\omega_m} \right) + \frac{F_2}{\eta_2} \left( \frac{V_2}{\omega_m} \right) + \dots + \frac{F_n}{\eta_n} \left( \frac{V_n}{\omega_m} \right)$$

## 2. Steady State Stability:

When Motor torque equals to load torque, equilibrium speed of a motor load can be achieved. At this speed, electric drive system operates in steady state.

Also during Transient Conditions, electrical motor is assumed to be in electrical equilibrium state. Steady state speed torque curves holds good for transient condition. The electrical time constant of motor is negligible in comparison with mechanical time constant for many electrical drives.



When small disturbances ( $T > T_L$ ) & ( $T_L > T$ ) the point A operates equilibrium point A.

Condition for stability  $\therefore \frac{dT_L}{d\omega_m} > \frac{dT}{d\omega_m}$

$\Delta\omega_m$  - Small perturbation in speed

$\Delta T$  - Small perturbation in motor torque

$\Delta T_L$  - Small perturbation in load torque

$$T = T_L + J \frac{d\omega_m}{dt} \quad \text{--- (1)}$$

$$(T + \Delta T) = (T_L + \Delta T_L) + J \frac{d(\omega_m + \Delta\omega_m)}{dt} \quad \text{--- (2)}$$

$$T + \Delta T = T_L + \Delta T_L + J \frac{d\omega_m}{dt} + J \frac{d\Delta\omega_m}{dt} \quad \text{--- (3)}$$

Subtract (2) & (3)

$$\frac{J d\Delta\omega_m}{dt} = \Delta T - \Delta T_L \quad \text{--- (4)}$$

$$\therefore \Delta T = \frac{dT}{d\omega_m} (\Delta\omega_m) \quad \text{--- (5)}$$

$$\Delta T_L = \frac{dT_L}{d\omega_m} (\Delta\omega_m) \quad \text{--- (6)}$$

Substitute equation (5) & (6) into (4)

$$J \frac{d\Delta\omega_m}{dt} + \left[ \frac{dT_L}{d\omega_m} - \frac{dT}{d\omega_m} \right] \Delta\omega_m = 0$$

In first order linear differential equation  
At time  $t=0$ , Initial deviation in speed will be

$$\Delta\omega_m = (\Delta\omega_m)_0 \exp \left\{ -\frac{1}{J} \left[ \frac{dT_L}{d\omega_m} - \frac{dT}{d\omega_m} \right] t \right\}$$



### 3. Typical Load torque characteristics:

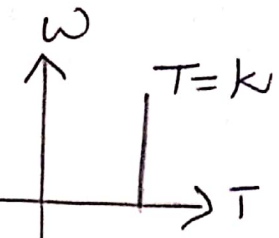
(i) Constant torque type

(i) Constant torque type ( $T=k$ )

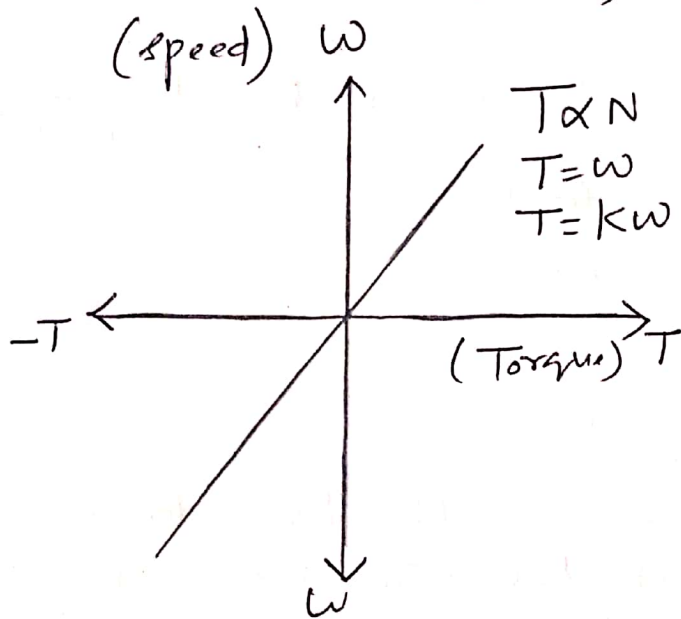
(ii) Torque proportional to speed (GENERATOR)

(iii) Torque proportional to square of the speed (FAN)

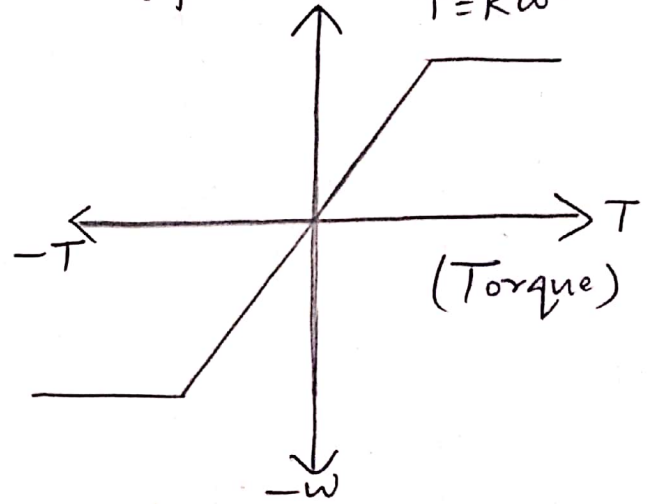
(iv) Torque Inversely proportional to speed (Constant POWER TYPE)



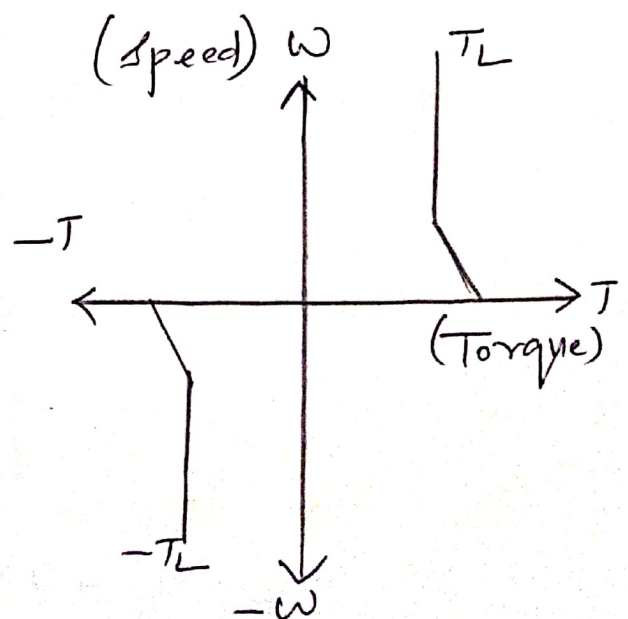
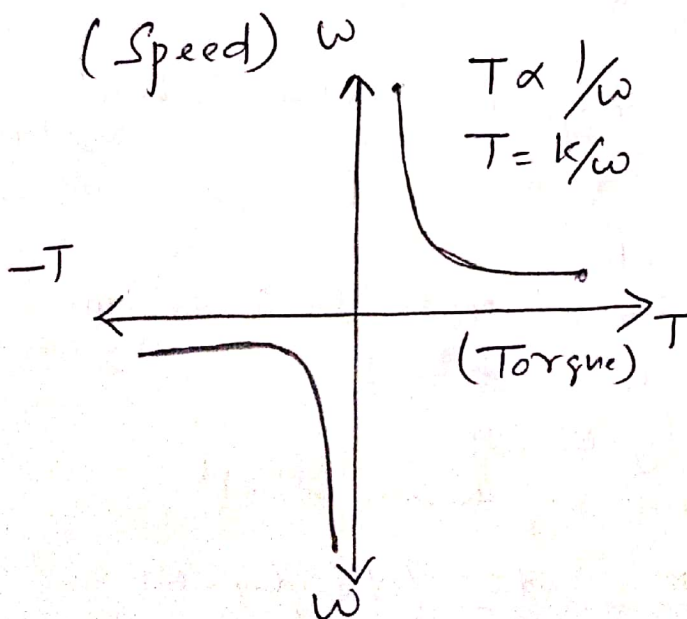
(ii) Torque proportional to speed: ( $T \propto \omega$ )



(iii) Torque proportional to square of speed ( $T \propto \omega^2$ )



(iv) Torque Inversely proportional to speed: ( $T \propto 1/\omega$ )



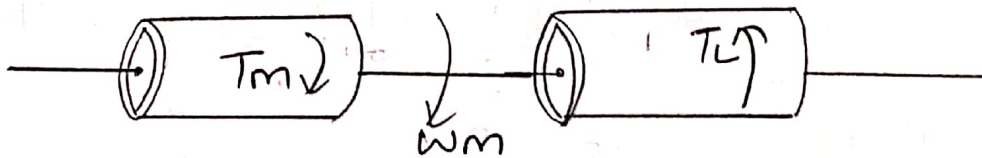


## 4. Selection of Electric drive:

- (i) Duty cycle, Speed Range, Speed Regulation, Efficiency, Nature of s-T characteristics, Speed fluctuations, Quantities of operation are the requirements for steady state operation
- (ii) Performance during starting, Braking, reversing Conditions and Acceleration & Deacceleration values are the requirements for transient operation.
- (iii) Source requirements  
AC/DC source, voltage magnitude, voltage fluctuations, line harmonics, I/p PF.
- (iv) Capital Cost, Running Cost, durability, Maintenance Needs.
- (v) Location and Environmental factors
- (vi) Weight and space Restrictions
- (vii) Reliability

## Fundamental Torque Equation:

The motor always rotates, but the load does not undergo rotational motion all the time. The load may undergo a translational motion or either undergo a rotational motion.



Fundamental Torque Equation Motor - Load System

$$T_m = T_L + \frac{d(J\omega_m)}{dt}$$

- J - Polar moment of Inertia of motor load system  $\text{kg-m}^2$   
 $\omega_m$  - Angular velocity of motor shaft  
 $T_m$  - Torque developed by Motor  
 $T_L$  - Load torque developed by Motor shaft

$$T_m = T_L + J \frac{d\omega_m}{dt} + \omega_m \frac{dJ}{dt} \quad \left( \text{For Constant Inertia } \frac{dJ}{dt} = 0 \right)$$

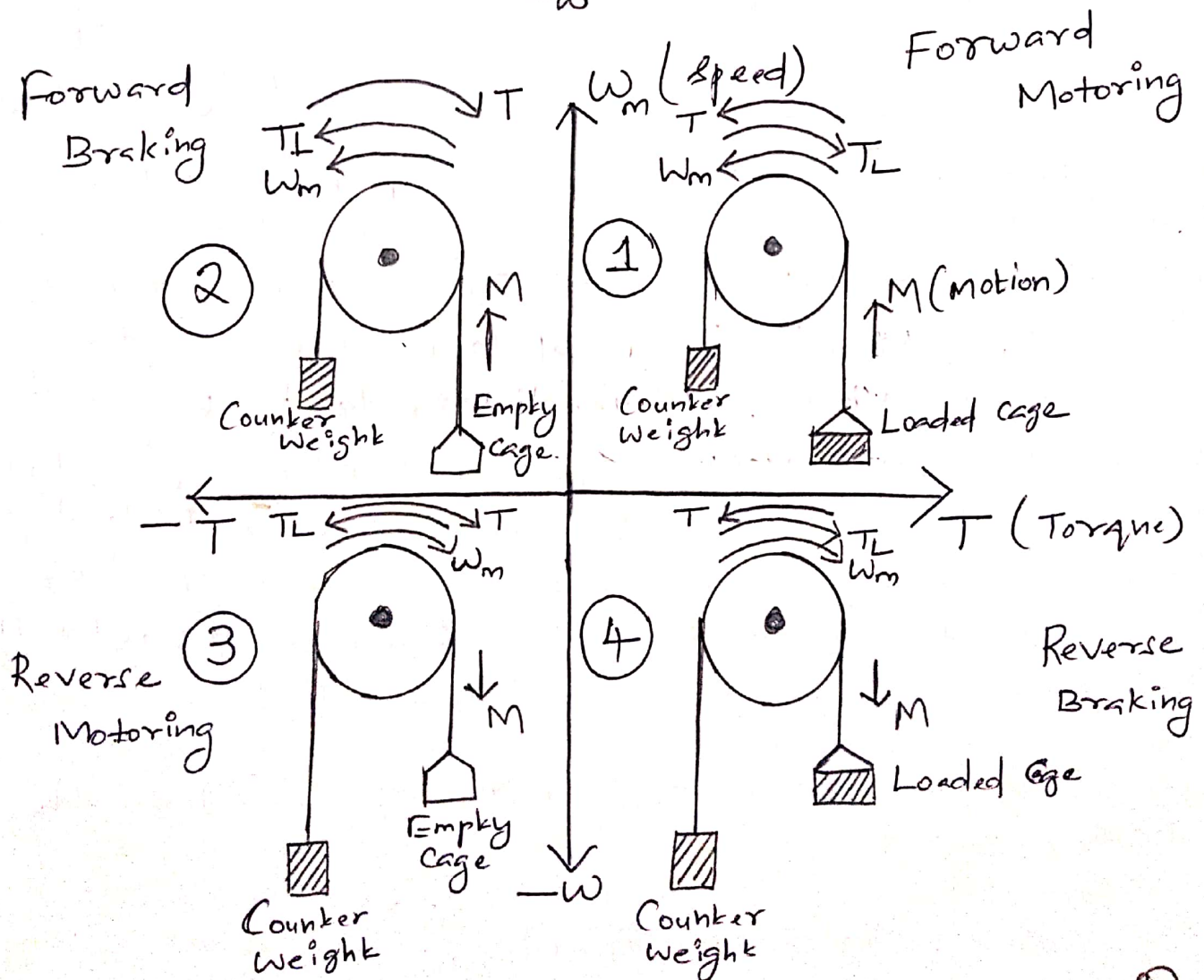
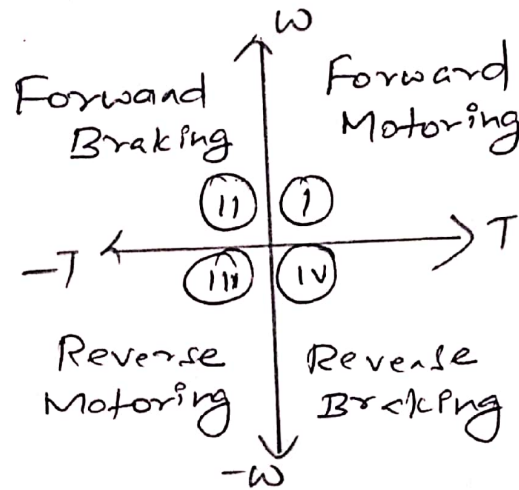
$$T_m = T_L + J \frac{d\omega_m}{dt}$$

# Multi Quadrant operation of drive System:

Speed and torque characteristics of electrical drive system.

In Motoring Mode - Electrical Energy Converted to Mechanical Energy

In Braking Mode - Mechanical Energy Converted to Electrical Energy





Power developed by Motor = Speed  $\times$  Torque

I<sup>st</sup> Quadrant - Forward Motoring - Power developed positive, Speed positive

II<sup>nd</sup> Quadrant - Forward Braking - Power Negative, Speed positive

III<sup>rd</sup> Quadrant - Reverse Motoring - Power positive, Speed Negative

IV<sup>th</sup> Quadrant - Reverse Braking - Power and Speed Negative

Let us consider the operation of a Hoist.  
A Hoist comprises of rope wound on a drum which is coupled to the shaft of the motor.

Note:

Weight of the Counter weight  $>$  Weight of Empty Cage

Weight of the Counter weight  $<$  Weight of a Fully Loaded cage

b. A Motor drives two loads. one has rotational motion. It is coupled to motor through a reduction gear with  $a=0.1$  and efficiency 90%. The load has moment of inertia of  $10 \text{ kg-m}^2$  & torque of  $10 \text{ Nm}$ . other load has translational motion and consists of  $1000 \text{ kg}$  weight to be lifted upto a uniform speed of  $1.5 \text{ m/s}$ . Coupling b/w the load and motor has an efficiency of 85%. Motor has inertia of  $0.2 \text{ kg-m}^2$  and runs at a constant speed of  $1420 \text{ rpm}$ . Determine equivalent Inertia referred to the motor shaft and power developed by the motor.

Total Moment of Inertia referred to the Motor shaft.

$$J = J_0 + a_1^2 J_1 + M_1 \left( \frac{v_1}{\omega_m} \right)^2$$



$$J_0 = 0.2 \text{ kg-m}^2, a_1 = 0.1, J_1 = 10 \text{ kg-m}^2$$

$$n = 1.5 \text{ m/s}$$

$$\omega_m = \frac{1420 \times \pi}{30} = 148.7 \text{ rad/s}$$

$$J = 0.2 + (0.1)^2 \times 10 + 1000 \left( \frac{1.5}{148.7} \right)^2$$

$$J = 0.4 \text{ kg-m}^2$$

$$T_L = \frac{a_1 T_{L1}}{n_1} + \frac{F_1}{\eta_1} \left( \frac{v_1}{\omega_m} \right)$$

$$\eta_1 = 0.9, \eta_1' = 0.1$$

$$T_{L1} = 10 \text{ Nm}, \eta_1' = 0.85$$

$$F_1 = 1000 \times 9.81, n_1 = 1.5 \text{ m/s}$$

$$\omega_m = 148.7 \text{ rad/s}$$

$$T_L = \frac{0.1 \times 10}{0.9} \times \frac{1000 \times 9.81}{0.85} \left( \frac{1.5}{148.7} \right)$$

$$T_L = 117.53 \text{ Nm}$$

$$\text{Power developed} = T_L \omega_m = 117.53 \times 148.7$$

$$P = 17.48 \text{ kW}$$

7. A drive has following parameters

$$T = 150 - 0.1N, \text{ Nm}$$

$$T_L = 100 \text{ Nm}$$

Initially the drive is operating in steady state. The characteristics of the load torque are changed  $T_L = -100 \text{ Nm}$ , calculate initial and final Equilibrium Speeds.

(i) Initial Equilibrium Speed :

For Steady State

$$T - T_L = 0$$

$$150 - 0.1N - 100 = 0$$

$$0.1N = 50$$

$$N = \frac{50}{0.1} = 500 \text{ rpm} \quad \therefore N = 500 \text{ rpm.}$$

(ii) final Equilibrium speed :

Characteristics of Load torque changed

$$150 - 0.1N - (-100) = 0$$

$$150 - 0.1N + 100 = 0$$

$$250 - 0.1N = 0$$

$$N = \frac{250}{0.1} = 2500 \text{ rpm}$$

$$\therefore N = 2500 \text{ rpm}$$