Department of petroleum Engineering

CH 8591 Heat Transfer

Unit - II

(1) By dimensional analysis derive the relationship between the dimensional numbers for natural convection.

Consider the case of natural / free convection from a vertical plane wall to an adjacent fluid.

The free convection heat transfer would depend on the same variables as in forced convection except the velocity 'U' because it is not caused by an external agency here.

Instead the fluid circulation is caused by a buoyant force which comes into play because of density variation due to temp. difference.

If 'ρ' is the bulk fluid, ρ_0' the fluid density inside the heated layer & 'ΔT' is the temp. diff b/w the heated fluid & the bulk value, then the coefficient of thermal expansion 'β' is defined as.

$$\rho = \rho_0 (1+\beta\Delta T)$$

The buoyant force,

$$F_b = (\rho - \rho_0) \cdot g$$

(or)

$$F_b = \beta g \cdot \rho_0 \cdot \Delta T$$

Hence, in free convection the variable 'U' is replaced by the variable ΔT, β and g

The list of significant variables along with their symbols & dimensions is given below.

Variables	Symbols	Dimension
Fluid density	'ρ'	ML^{-3}
viscosity	μ	$ML^{-1}t^{-1}$
Heat capacity	C_p	$L^2 t^{-2} T^{-1}$
Thermal conductivity	k	$MLt^{-3}T^{-1}$
Coeff. of thermal expansion	β	T^{-1}
Gravitational acceleration	g	Lt^{-2}
Temp. difference	ΔT	T
Length	L	L
HT - coeff	h	$Mt^{-3}T^{-1}$

Out of these variables, normally 'β' and g are taken together as 'βg' with dimension $LT^{-1}t^{-2}$.

So, according to Buckingham's π theorem, the no of independent dimensionless groups formed in $8-4=4$.

If we designate L, ρ, μ & k as the core variable the π groups can be formed as below

$$\pi_1 = L^a \rho^b \mu^c k^d \Delta T$$
$$\pi_2 = L^e \rho^f \mu^i k^j \beta g$$
$$\pi_3 = L^l \rho^m \mu^n k^o C_p$$
$$\pi_4 = L^p \rho^q \mu^r k^s h$$

Following the procedure of equating the exponents of M, L, T, t to zero, we can finally show that

$$\pi_1 = \frac{L^2 \rho^2 k \Delta T}{\mu^3}$$

$$\pi_2 = \frac{L \mu \beta g}{k}$$

$$\pi_3 = \frac{\mu C_p}{k} = Pr$$ prandtl Number (prandtl)

$$\pi_4 = \frac{hL}{k} = Nu$$ Nusselt Number

Several experimental & analytical studies have shown that the first two groups always appear together as a single dimensional group.

The combined parameter so formed is called Grashof number.

$$\boxed{Gr = \pi_1 \cdot \pi_2 = \frac{\rho^2 \beta g L^3 \Delta T}{\mu^2}}$$

The free / Natural convection heat transfer data be represented in the dimensional form as

$$\boxed{Nu = f\,(Gr \cdot Pr).}$$

(2) a) By dimensional analysis prove that $Nu = f(Re \cdot Pr)$ for forced convective heat transfer operation.

b) Explain the physical significance of the above mentioned groups.

$\boxed{\text{Nusselt Number}}$

$$Nu = \frac{hL}{k}$$

h - overall heat transfer coeff

L - Characteristic length

k - thermal conductivity

It is the ratio of wall temp gradient to the temp. gradient across the fluid in the pipe

$\boxed{\text{Reynolds number}}$

$$Re = \frac{l v \rho}{\mu}$$

(or)

$$Re = \frac{D v \rho}{\mu}$$

l - characteristic length

v - velocity of fluid

ρ - density

μ - viscosity

It is defined as ratio between Inertia to viscous force

$\boxed{\text{prandtl number}}$

$$Pr = \frac{C_p \mu}{k} \quad \text{also} = \frac{\nu}{\alpha}$$

Cp - specific heat

μ - viscosity

k - Thermal conductivity

It is defined as momentum diffusivity to thermal diffusivity.

$\boxed{\text{Stanton number}}$

$$St = \frac{Nu}{Re \cdot Pr} \Rightarrow \frac{hD/k}{\frac{Dv\rho}{\mu} \times \frac{C_p \mu}{k}} = \frac{h}{C_p \rho u}$$

Ratio of rate of wall heat transfer by convection to rate of heat transfer by bulk flow.

Peclet Number

$$Pe = Re \cdot Pr = \frac{DVP}{\mu} \cdot \frac{C_p \cdot \mu}{k} = \frac{\rho V D C_p}{k}$$

ratio of, rate of heat transfer by bulk flow to rate of heat transfer by conduction

Graetz Number

$$Gz = Pe \cdot \frac{d}{L} = Re \cdot Pr \cdot \frac{d}{L}$$

This is similar to peclet number but used in connection with analysis of heat transfer in laminar flow in pipes.

Grashof number

$$Gr = \frac{\beta g L^3 \Delta T}{\nu^2}$$

ie ratio between Buoyancy force to viscous force

$\Delta T = T_s - T_0$

g - Acceleration due to gravity

β - coeff of volumetric expansion

T_s - surface temp

T_0 - Ambient Temp

ν - viscous force

L - Length

Biot Number

$$Bi = \frac{h L_c}{k}$$

It is the ratio of internal thermal resistance of a solid body to its surface thermal resistance.

Note: Gr. Number plays the same role in natural convection as the Reynolds number does in forced convection.

③ ~~a. Explain the physical significance of the~~

④. Explain in detail about analogies between transfer of momentum and heat (at least three analogies)

Momentum & Heat Transfer Analysis

1. Reynolds Analogy

$$\frac{Nu}{Re \cdot Pr} = St = \frac{h}{\rho C_p u} = f/2$$

This can be used to determine the heat transfer coefficient 'h' if the friction factor 'f' is known.

2. Prandtl ~~[illegible]~~ Analogy

$$St = \frac{f/2}{1 + 5\sqrt{f/2}\,(Pr - 1)}$$

Here 'f' is fanning friction factor. It provides a more realistic eqn of turbulent flow.

It reduced to Reynolds analogy if $Pr = 1$.

3. Chilton - Colburn Analogy

$$\frac{Nu}{Re \cdot Pr^{1/3}} = j_H = f/2$$

where, j_H is Colburn j - factor

using the well known correlation for the friction factor, $\boxed{f = 0.046\ Re^{-0.2}}$ for pipe flow

j factor is given by $\boxed{j_H = 0.023\ Re^{-0.2}}$

i) Free convection from a flat plate (Eqn for Natural convection)

Churchill - chu equation.

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.825 + \frac{0.387\,(Ra_L)^{1/6}}{\left\{1+\left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{8/27}}$$

for Laminar flow (ie $Ra_L < 10^9$)

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.68 + \frac{0.67\,(Ra_L)^{1/4}}{\left\{1+\left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{4/9}}$$

where, Ra_L - Rayleigh number.

ii) free convection from a cylinder

Churchill - chu equation

$$\overline{Nu} = \frac{\bar{h}d}{k} = \left[0.60 + \frac{0.387\,(Ra_d)^{1/6}}{\left\{1+\left(\frac{0.559}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

The condition is

$$10^{.5} < Ra_d < 10^{12}$$

Note: Rayleigh no. $\boxed{Ra = Gr \times Pr}$

iii) free convection from a sphere : churchill eqn

$$\overline{Nu} = \frac{\bar{h}d}{k} = 2 + \frac{0.589\,(Ra_d)^{1/4}}{\left\{1+\left(\frac{0.469}{Pr}\right)^{9/16}\right\}^{4/9}}; \quad Pr \geq 0.7, \; Ra_d \leq 10^{11}$$

(5) Derive the correlation for forced convection using dimensional analysis.

Consider the case of a fluid flowing across a heated tube.

The various variables pertinent to this problem along with their symbols and dimensions are given in table below.

Variable	Symbol	Dimension
Fluid density	ρ	ML^{-3}
Tube diameter	D	L
Velocity	U	Lt^{-1}
Viscosity	μ	$ML^{-1}t^{-1}$
Specific heat	C_p	$L^2t^{-2}T^{-1}$
Thermal conductivity	k	$MLt^{-3}T^{-1}$
Heat transfer coefficient	h	$Mt^{-3}T^{-1}$

From the table, we see that there are seven variables and four basic dimensions.

According to Buckingham's theorem, the no. of independent dimensionless groups that can be formed is $= 7 - 4 = 3$

Let these three dimensionless groups be symbolised by π_1, π_2 & π_3

"Each dimensionless parameter will be formed by combining a core group of 'r' variables with one of the remaining variables not in the core"

In this case, the core will include any 4 of the variables, which among them, include all of the basic dimensions.

We may, arbitrarily choose $D, \rho, \mu,$ & k as the core.

The groups to be formed are now represented as the following 'π' groups.

$$\pi_1 = D^a \rho^b \mu^c k^d u$$

$$\pi_2 = D^e \rho^f \mu^g k^h c_p$$

$$\pi_3 = D^j \rho^l \mu^m k^n h$$

Since these groups are to be dimensionless, the variables are raised to certain exponents $a, b, c \ldots m, n$

Starting with π_1 we write dimensionally as:

$$M^0 L^0 T^0 t^0 = 1 = 1 = (L)^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{ML}{t^3 T}\right)^d \left(\frac{L}{t}\right)$$

Equating the sum of the exponents of each basic dimension to zero, we get the following set of equations.

for,

M: $\quad 0 = b + c + d$

L: $\quad 0 = a - 3b - c + d + 1$

t: $\quad 0 = -c - 3d - 1$

T: $\quad 0 = -d$

Solving these equations, we get:

$$\left.\begin{array}{l} d = 0 \\ c = -1 \\ b = 1 \\ a = 1 \end{array}\right\} \text{giving} \quad \pi_1 = \frac{D u \rho}{\mu} = Re_D \text{ (Reynolds number)}$$

Similarly for π_2

$$1 = (L)^e \left(\frac{M}{L^3}\right)^f \left(\frac{M}{Lt}\right)^g \left(\frac{ML}{t^3T}\right)^i \left(\frac{L^2}{t^2T}\right)$$

for

$$M: \quad 0 = f+g+i$$

$$L: \quad 0 = e-3f-g+i+2$$

$$t: \quad 0 = -g-3i-2$$

$$T: \quad 0 = -i-1$$

From these, we find

$$\left.\begin{array}{l} i = 1 \\ g = 1 \\ f = 0 \\ e = 0 \end{array}\right\} \text{giving} \quad \pi_2 = \frac{\mu C_p}{k} = Pr$$

(Prandtl number)

By following a similar procedure, we can obtain

$$\pi_3 = \frac{hD}{k} = Nu \quad \text{(Nusselt Number)}$$

we may now express

$$F(\pi_1, \pi_2, \pi_3) \text{ as } \quad Nu = \phi\,(Re, Pr)$$

<u>Note:</u> Here we chose the core variable quite arbitrarily.

Had we choosen a different core group, we would have got Stanton number

$$St = \frac{h}{\rho v C_p} \quad \text{Also,} \quad St = \frac{Nu}{Re \cdot Pr.}$$

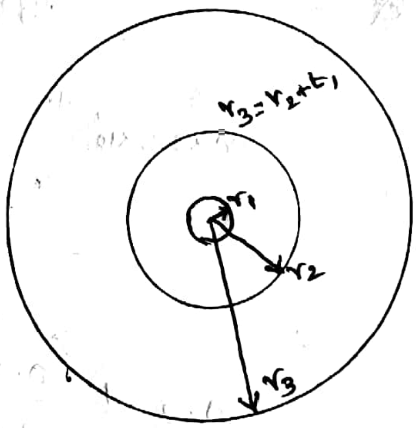
So, another form of correlating heat transfer data is

$$St = \phi\,(Re, Pr).$$

Thus dimensional analysis has shown us a way to reduce the 7 significant variables of forced convection to three dimensionless parameters.

(3) Steam at 120°C is flowing through at wrought-iron (k = 59 W/mK) tube of ID = 5cm and OD = 7cm which is covered with 1 cm thick asbestos (k = 0.1105 W/mK) insulation. If the convection heat transfer coeff at the inner and outer surfaces of tube are 200 and 10 W/m²K respectively, and the atmospheric air is 25°C. Estimate the rate of heat losses from steam per meter length of tube.

Assume that the steam in the tube is held at a constant temperature.



Given

$T_a = 120°C$

$= 120 + 273$

$= 393\ K$

$k_1 = 59\ W/mK$; $k_2 = 0.1105\ W/mK$

$ID \Rightarrow D_1 = 5\ cm$

$r_1 = 2.5 \times 10^{-2}\ m$

$OD \Rightarrow D_2 = 7\ cm$

$r_2 = 3.5 \times 10^{-2}\ m$

$t_1 = 1\ cm \Rightarrow 1 \times 10^{-2}\ m.$

$h_a = 200\ W/m^2K$

$h_b = 10\ W/m^2K$

$T_b = 25 + 273 \Rightarrow 298\ K$

$r_3 = r_2 + t_1 \Rightarrow 3.5 \times 10^{-2} + 1 \times 10^{-2} \Rightarrow 4.5 \times 10^{-2}\ m$

329.7

To Find

$Q = ?$

Take $L = 1\,m$

Formula to be used

$$Q = \frac{\Delta T}{R}$$

$$= \frac{T_a - T_b}{\frac{1}{h_a r_1} + \frac{\ln(r_2/r_1)}{2\pi k_1 L} + \frac{\ln(r_3/r_2)}{2\pi k_2 L} + \frac{1}{h_b r_3}}$$

$$Q = \frac{393 - 298}{\frac{1}{200 \times 2.5\times10^{-2}} + \frac{\ln(35/25)}{2\pi \times 59 \times 1} + \frac{\ln(45/35)}{2\pi \times 0.1105 \times 1} + \frac{1}{10 \times 4.5\times10^{-2}}}$$

$$= \frac{95}{0.2 + \frac{0.3364}{370.70} + \frac{0.2513}{0.6942} + \frac{1}{2.222}}$$

$$= \frac{95}{0.2 + 9.07\times10^{-4} + 0.361 + 2.222}$$

$$\boxed{Q = 33.64\ W}$$

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