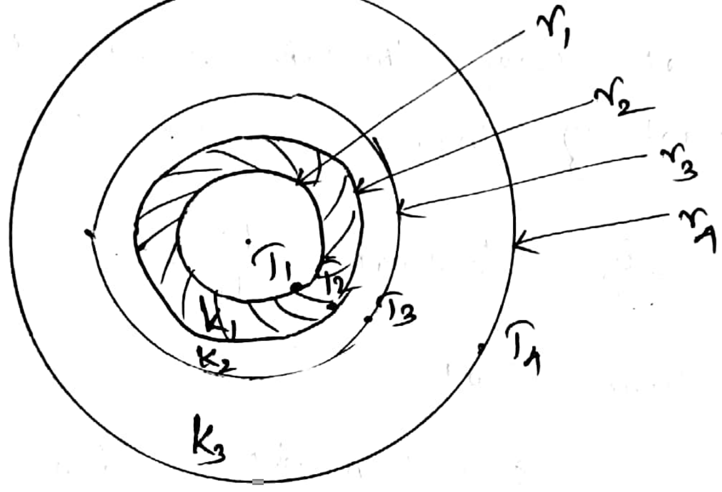
Dept. of Petroleum Engg.

B. Tech Petroleum Engineering

II / IV Unit - I CH - 8591 Heat Transfer

Conduction

Q. No. ① Derive the steady state heat conduction equation in a composite cylinder comprising three layers.



Let the hollow composite cylinder be made out of three different materials 1, 2 & 3.

Let k_1 k_2 k_3 be the thermal conductivities of materials.

Let r_1 r_2 r_3 & r_4 are radius

Let 'L' be the length of material

Temp. drop across layers be ΔT_1, ΔT_2 & ΔT_3 respectively.

Let ΔT - overall Temp. drop

$\Delta T_1 = T_1 - T_2$; $\Delta T_2 = \Delta T_2 - T_3$; $\Delta T_3 = T_3 - T_4$

Let, T_1 - Temp inside the cylinder

T_4 - Temp. outside the cylinder

T_2 & T_3 - Interface Temperature

Let 'A' be the area of the cylinder '$2\pi r L$'

$L(2\pi r L)$

Rate of heat flow through layor ie, through the material of thermal conductivity k_1 is given by

$$Q_1 = -k \, 2\pi r L \left(\frac{dT}{dr}\right) \text{ ——— } (1)$$

we know that

$$Q = -kA \cdot \frac{dT}{dr}$$ from Fourier's law

Re arrange the eqn (1)

$$\frac{dr}{r} = \frac{-k(2\pi L)}{Q_1} dT \text{ ——— } (2)$$

Integrate the eqn (2) we get,

Limits where $r = r_1$, $T = T_1$

$r = r_2$ $T = T_2$

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{-k \, 2\pi L}{Q_1} \int_{T_1}^{T_2} dT \text{ ——— } (3)$$

$$\ln [r]_{r_1}^{r_2} = \frac{-k_1 2\pi L}{Q_1} [T]_{T_1}^{T_2}$$

$$\ln [r_2 - r_1] = \frac{-k_1 2\pi L}{Q_1} [T_2 - T_1]$$

$$\ln r_2 - \ln r_1 = \frac{k_1 2\pi L}{Q_1} [T_1 - T_2] \quad \text{--- (4)}$$

$$\ln \frac{r_2}{r_1} = \frac{k_1 2\pi L}{Q_1} (T_1 - T_2)$$

ie, $$T_1 - T_2 = \frac{Q_1 \ln\left(\frac{r_2}{r_1}\right)}{k_1 2\pi L} \quad \text{--- (5)}$$

where $\Delta T_1 = T_1 - T_2$

$$\therefore \Delta T_1 = \frac{Q_1}{k_1 (2\pi L) / \ln\left(\frac{r_2}{r_1}\right)} \quad \text{--- (6)}$$

Similary for layer (2) & (3)

$$\Delta T_2 = \frac{Q_2}{k_2 (2\pi L) / \ln(r_3/r_2)} \quad \text{--- (7)}$$

$$\Delta T_3 = \frac{Q_3}{k_3 (2\pi L) / \ln(r_4/r_3)} \quad \text{--- (8)}$$

(A) Adding eqn 6, 7 & (8)

we get.

$$\Delta T_1 + \Delta T_2 + \Delta T_3 = \frac{Q_1}{k_1 (2\pi L)/\ln(r_2/r_1)} + \frac{Q_2}{k_2(2\pi L)/\ln(r_3/r_2)} + \frac{Q_3}{k_3(2\pi L)/\ln(r_4/r_3)} \quad \text{—— (9)}$$

$$\Delta T = Q\left[\frac{1}{k_1 (2\pi L)/(\ln r_2/r_1)} + \frac{1}{k_2(2\pi L)/\ln(r_3/r_2)} + \frac{1}{k_3(2\pi L)/\ln(r_4/r_3)}\right] \quad \text{—— (10)}$$

wkt $\boxed{Q_1 + Q_2 + Q_3 = Q}$

$$\therefore Q = \frac{\Delta T}{\frac{\ln(r_2/r_1)}{k_1 2\pi L} + \frac{\ln(r_3/r_2)}{k_2 2\pi L} + \frac{\ln(r_4/r_3)}{k_3 2\pi L}} \quad \text{—— (11)}$$

$$Q = \frac{2\pi L\, \Delta T}{\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{\ln(r_4/r_3)}{k_3}} \quad \text{—— (12)}$$

Let R_1, R_2 & R_3 be the thermal resistance offered by lay 1, 2 & (3)

$$R_1 = \frac{\ln(r_2/r_1)}{k_1 2\pi L} ; \quad R_2 = \frac{\ln(r_3/r_2)}{k_2 2\pi L} ; \quad R_3 = \frac{\ln(r_4/r_3)}{k_3 2\pi L}$$

eqn (12) becomes.

$$Q = \frac{\Delta T}{R_1 + R_2 + R_3}$$

$$\boxed{Q = \frac{\Delta T}{R}}$$

(2) A furnace wall has three layers. The inner layer of 11 cm thickness is made of fire brick ($k = 1.08$ W/mK) & intermediate layer is made up of masonry brick of thickness 9cm ($k = 0.72$ W/mK) followed by a 6cm thick concrete wall ($k = 1.427$ W/mK). When the furnace is continuous operation, the inner surface of the wall is at 725°C. while the outer surface is @ 38°C compute

i) The rate of heat loss per unit area of wall

ii) Temp at the interface of firebrick & masonry brick

iii) The temp at the interface of the masonry brick & concrete.

Given

$x_1 = 11cm = 11 \times 10^{-2} m$

$k_1 = 1.08$ W/mK

$x_2 = 9cm = 9 \times 10^{-2} m$

$k_2 = 0.72$ W/mK

$x_3 = 6 \times 10^{-2} m$

$k_3 = 1.427$ W/mK

$T_1 = 725°C = 998 K$

$T_4 = 38°C = 311 K$

To find:

$Q = ?$

$T_2 = ?$

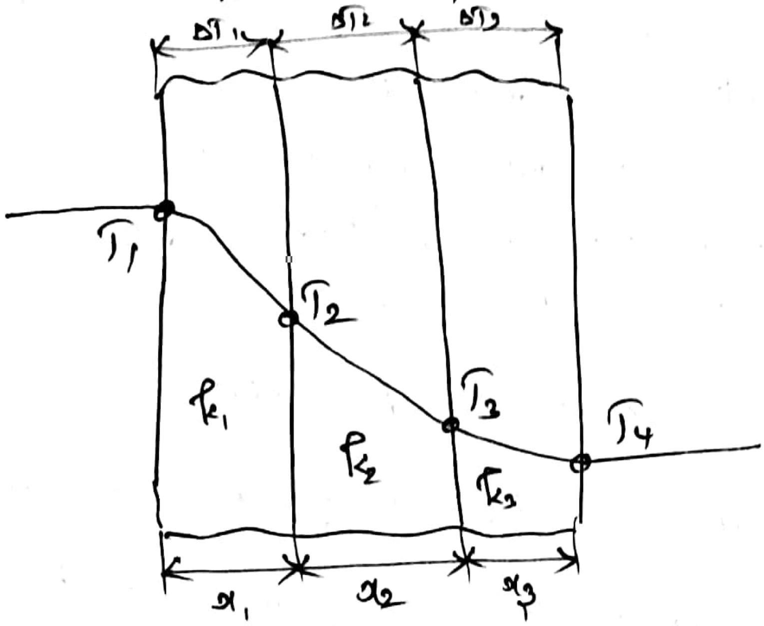
$T_3 = ?$

⑥

Formula Required

$$Q = \frac{\Delta T}{R} = \frac{\Delta T_1 + \Delta T_2 + \Delta T_3}{R_1 + R_2 + R_3}$$

86 TN



$$Q = \frac{998 - 311}{\frac{11\times10^{-2}}{1.08} + \frac{9\times10^{-2}}{0.72} + \frac{6\times10^{-2}}{1.427}}$$

i.e. $\boxed{R = x/_{k,A}}$

$\boxed{\Delta T_1 = T_1 - T_2}$

$$= \frac{687}{0.10 + 0.125 + 0.042}$$

$$= 687 / 0.267$$

$$\boxed{Q = 2573\,W}$$

WKT, At steady state

$$Q_1 = Q_2 = Q_3 = 'Q'$$

$$\boxed{Q_1 = \frac{T_1 - T_2}{x_1/k_1 A}}$$

$$2573 = \frac{998 - T_2}{11 \times 10^{-2} / 1.08}$$

$$2573 = \frac{998 - T_2}{0.10}$$

$$\boxed{T_2 = 740.7\,K}$$

$$Q_2 = \frac{T_2 - T_3}{a_2 / k_2 A}$$

$$2573 = \frac{7407 - T_3}{9 \times 10^{-2} / 0.72}$$

$$2573 = \frac{740.7 - T_3}{0.125}$$

$$\boxed{T_3 = 648.1\,K}$$

Q & T_2 & T_3 are found.

8 (3) i) Explain the different types of Extended surface used for Heat Transfer.

✓ By increasing the surface area in contact with air or providing fins.

✓ By increasing the heat transfer coefficient for the surface.

✓ By increasing the Temp of the hot surface or by increasing the Temp diff b/w hot & cold bodies

whenever the available surface is found inadequate transfer the required quantity of heat with available Temp drop & convective HT coeff, extended surfaces or fins are used.

The fins are designed & manufactured in many shapes & forms.

They manufactured in different geometries depending upon the practical applications.

Types of fins:

✓ Straight fin of uniform cross section

✓ Straight fin of nonuniform cross section

✓ Annular fin

✓ pin fin

major categories of extended surface heat exchangers are

Tube-fin

Fins are generally used to enhance the heat transfer from a given surface.

3.57. Describe the measurement of thermal conductivity with neat diagram.

There are a no. of possibilities to measure thermal conductivity, each of them suitable for limited range of materials, depending on the thermal properties and the medium temp.

There are two basic techniques,

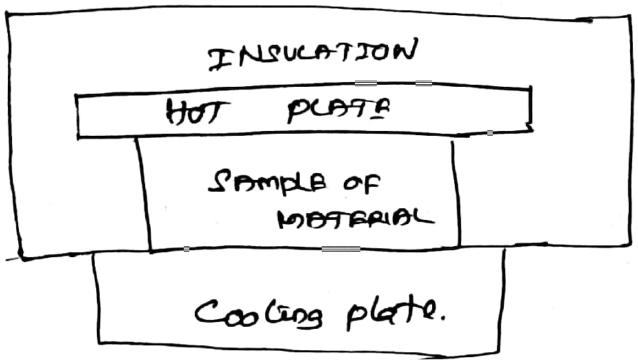
- Steady state
- Unsteady state

Steady state: techniques performs a measurement when material that is analysed is in complete equilibrium. This makes the process of signal analysis very easy.

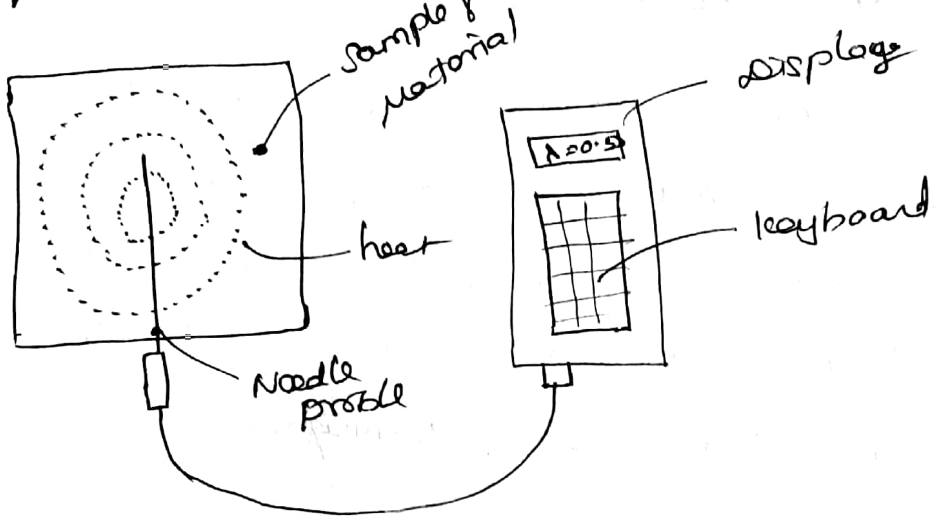
Non steady state techniques performs a measurement during the process of heating up.

There are many types of Instruments available to measure thermal conductivity.

- Guarded hot plate, Hot wire, modified hot wire Laser flash diffusivity.



Hot wire, modified Hot wire, laser flash diffusivity.



Determining of Thermal Conductivity in steady state.

$$\lambda = \frac{q \times d}{T_1 - T_2} \quad (W/mK)$$

where,

q - quantity of heat passing through a unit area of the sample in unit time [W/m^2]

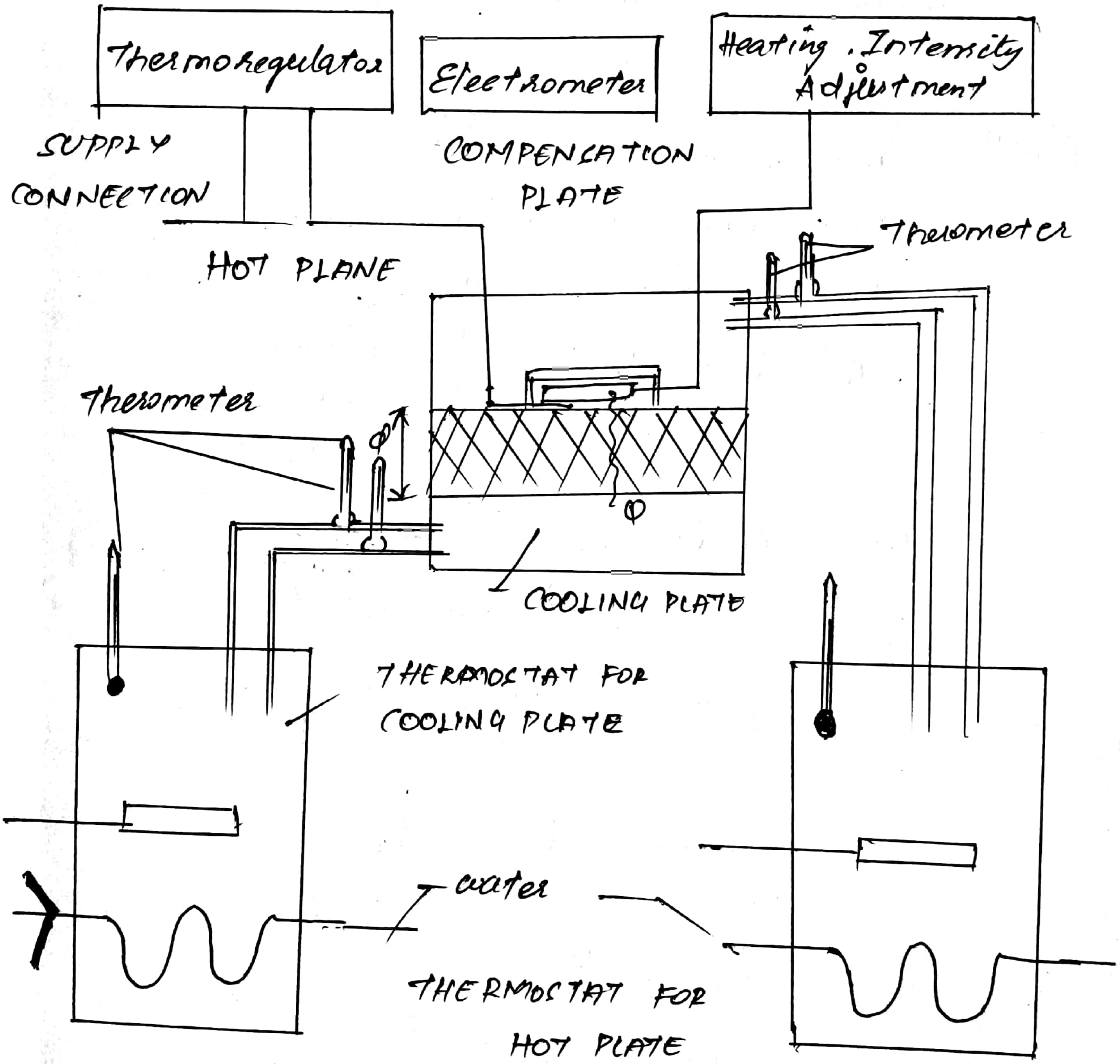
d - Distance (m)

T_1 - Temp on warmer side [K]

T_2 - Temp on colder [K]

$$q = \frac{Q}{A} \quad W/m^2$$

Temp in the laboratory during the measuring should not vary more than $\pm 2°C$ and relative humidity should not exceed 65%.



(4.) A furnace is constructed with 225 mm of firebrick, 120 mm of insulating brick and 225 mm of building brick. The inside temperature is 1200 K and outside temp is 330 K. If the thermal conductivities are 1.4, 0.2 and 0.7 W/mK, find the heat loss per unit area and the temperature at the junction of the firebrick and insulating brick.

Given

$x_1 = 225 \text{ mm} \Rightarrow 225 \times 10^{-3} \text{ m}$

$x_2 = 120 \text{ mm} \Rightarrow 120 \times 10^{-3} \text{ m}$

$x_3 = 225 \text{ mm} \Rightarrow 225 \times 10^{-3} \text{ m}$

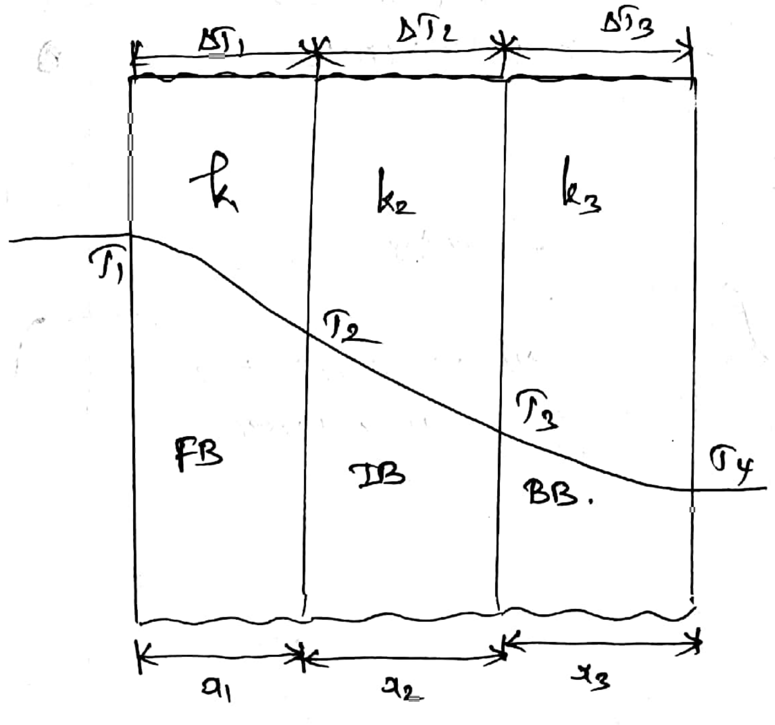
$T_1 = 1200 \text{ K}$, $T_4 = 330 \text{ K}$, $k_1 = 1.4 \text{ W/mK}$

$k_2 = 0.2 \text{ W/mK}$

$k_3 = 0.7 \text{ W/mK}$

To find:

$Q = ?$ & $T_2 = ?$



13.

$$Q = \frac{T_1 - T_4}{x_1/k_1 A + \frac{x_2}{k_2 A} + \frac{x_3}{k_3 A}} \Rightarrow \frac{\Delta T}{R_1 + R_2 + R_3}$$

$$Q = \frac{1200 - 330}{\frac{225 \times 10^{-3}}{1.4} + \frac{120 \times 10^{-3}}{0.2} + \frac{225 \times 10^{-3}}{0.7}}$$

$$Q = \frac{810}{0.16 + 0.6 + 0.031}$$

$$\boxed{Q = 813\ W}$$

WKT,

$$Q = Q_1 = Q_2 = Q_3$$

$$Q = \frac{T_1 - T_2}{x_1/k_1 A}$$

$$= \frac{1200 - T_2}{225 \times 10^{-3}/1.4}$$

$$813 = \frac{1200 - T_2}{0.16}$$

$$\boxed{T_2 = 1070\ k}$$

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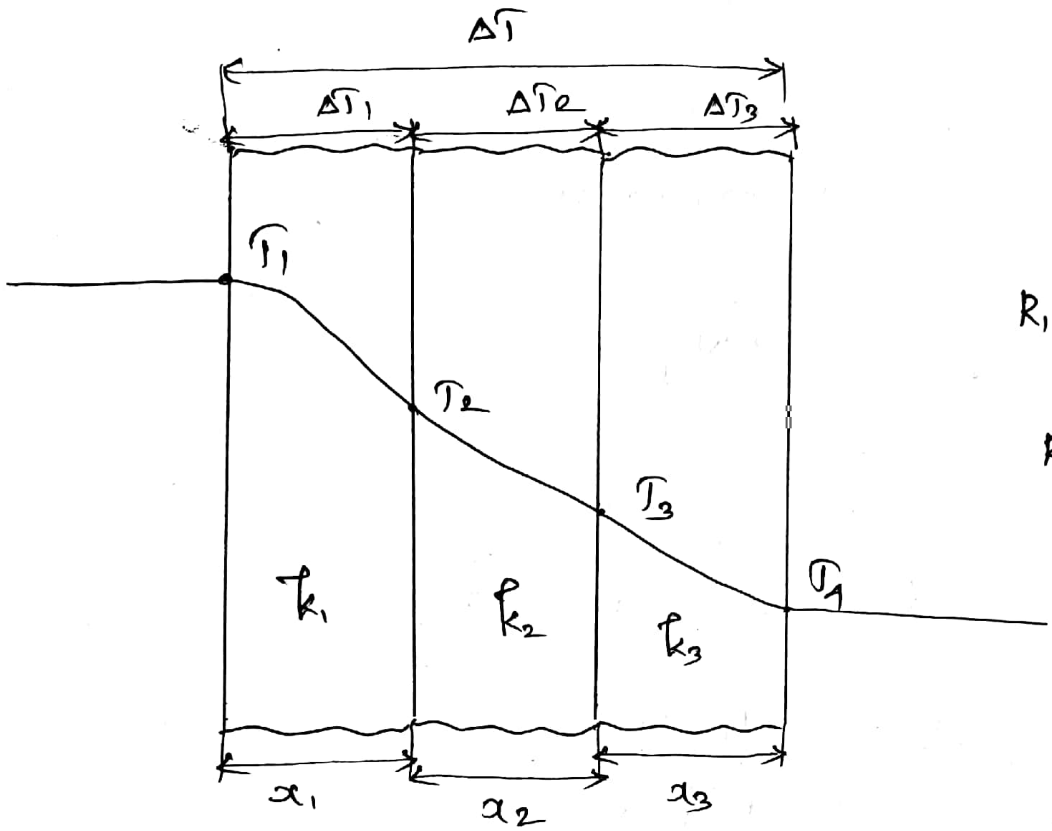
5. Derive an expression for the rate of heat transfer through a composite plane wall consisting of heterogeneous layers having thermal conductivities k_1, k_2 and k_3 respectively.



$R_1 = x_1 / k_1 A$

$R_2 = x_2 / k_2 A$

$R_3 = x_3 / k_3 A$.

Let T_1, T_4 are the inlet and outlet temperature of the body

T_2, T_3 are the interface temp.

k_1, k_2, k_3 be the thermal conductivities of material 1, 2 & 3 respectively

x_1, x_2, x_3 – thickness (distance) of the three layers.

$\Delta T_1, \Delta T_2, \Delta T_3$ - Temp drop b/w the layers

ΔT - overall Temp drop

WKT $\Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3$. ——— (1)

Three different types of material connected in series is known as composite wall.

Generally Heat transfer from high temp region to low temp area.

Eqn (1) is desired to derive an equation gives the rate of heat flow through series of resistance.

Layer (1)

$$Q_1 = \frac{k_1 A (T_1 - T_2)}{x_1} \quad \text{———} \quad (2)$$

$$(T_1 - T_2) = \frac{Q_1}{k_1 A / x_1} \quad \text{———} \quad (3)$$

$$\Delta T_1 = \frac{Q_1}{k_1 A / x_1} \quad \text{———} \quad (4)$$

$\because \Delta T_1 = (T_1 - T_2)$

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Similarly for layer 2 & 3

$$\Delta T_2 = \frac{Q_2}{k_2 A / x_2} \quad \text{———} \quad (5)$$

$\Delta T_2 = T_2 - T_3$

$$\Delta T_3 = \frac{Q_3}{k_3 A / x_3} \quad \text{———} \quad (6)$$

$\Delta T_3 = T_3 - T_4$

Adding eqn (4) (5) & (6) we get,

$$\Delta T_1 + \Delta T_2 + \Delta T_3 = \frac{Q_1}{k_1 A / x_1} + \frac{Q_2}{k_2 A / x_2} + \frac{Q_3}{k_3 A / x_3} = \Delta T \quad \text{———} \quad (7)$$

under steady state cond^n of heat flow all the heat passing layer (1) must pass through 2 and in turn pass layer (3)

So, Q_1, Q_2, & Q_3 must be equal and can be denoted by Q.

Eqn (7) becomes

$$\frac{Q}{k_1 A/d_1} + \frac{Q}{k_2 A/d_2} + \frac{Q}{k_3 A/d_3} = \Delta T \quad \text{—— (8)}$$

$$Q\left[\frac{1}{k_1 A/d_1} + \frac{1}{k_2 A/d_2} + \frac{1}{k_3 A/d_3}\right] = \Delta T \quad \text{—— (9)}$$

$$Q = \frac{\Delta T}{\left[\frac{1}{k_1 A/d_1} + \frac{1}{k_2 A/d_2} + \frac{1}{k_3 A/d_3}\right]} \quad \text{—— (10)}$$

$$Q = \frac{\Delta T}{d_1/k_1 A + x_2/k_2 A + d_3/k_3 A} \quad \text{—— (11)}$$

Let R_1 R_2 R_3 be the thermal resistance offered by layer 1, 2, & 3.

$$R_1 = d_1/k_1 A$$

$$R_2 = x_2/k_2 A$$

$$R_3 = x_3/k_3 A$$

Eqn (11) becomes

$$Q = \frac{\Delta T}{R_1 + R_2 + R_3} \quad \text{--- (12)}$$

If 'R' is overall resistance, then for resistance in series we have

$$R = R_1 + R_2 + R_3$$

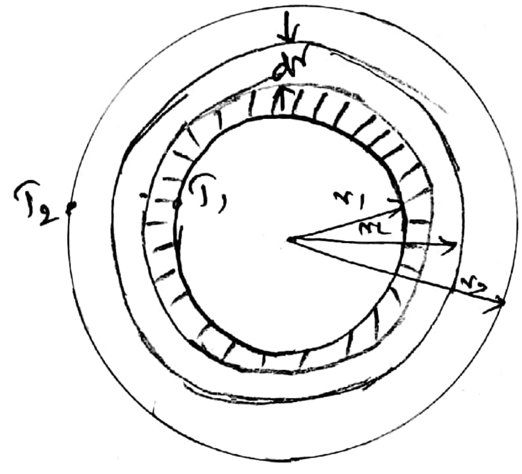
Eqn (12) becomes

$$\boxed{Q = \Delta T / R}$$

$$\text{Rate of Heat transfer} = \frac{\text{Temp. driving force}}{\text{Thermal resistance.}}$$

(b) Derive the steady. State heat conduction equation for cylindrical system.

Consider a thick walled hollow cylinder as shown in figure.



Heat flow through thick walled cylinder.

Let,

The inside radius of the cylinder is = 'r_1'

The outside radius of the cylinder is = 'r_2'

Length of the cylinder = 'L'

Thermal conductivity of the cylinder material = 'k'

The temperature of the inside surface is = 'T_1'

The temperature of the outside surface is = 'T_2'

Assume that $T_1 > T_2$, therefore heat flows from inside of cylinder to outside. Let 'A' be the area of the cylinder is $2\pi rL$. consider a very thin cylinder (cylindrical element), concentric with the main cylinder.

of radius r, where r is between r_1 and r_2. The thickness of wall of this cylindrical element is dr. The rate of heat flow at any radius r is given by.

$$Q = -k\,2\pi r L\left[\frac{dT}{dr}\right] \longrightarrow ①.$$

Equation (1) is similar to equation $\left(Q = -KA\left(\frac{dT}{dx}\right)\right)$,

Re arranging the equation (1), we get.

$$\frac{dr}{r} = -\frac{K2\pi L}{Q}\,dT \longrightarrow (2).$$

Only varibles in equation (2) are r and T.

Integrate the equation (2) between the limits when $r = r_1$, $T = T_1$ and when $r = r_2$, $T = T_2$.

$$\int_{r_1}^{r_2} \frac{dr}{r} = -\frac{K2\pi L}{Q}\int_{T_1}^{T_2} dT \longrightarrow (3).$$

$$\ln\left[r\right]_{r_1}^{r_2} = \frac{-k\,2\pi L}{Q}\left[T\right]_{T_1}^{T_2}$$

$$\ln\left[r_2 - r_1\right] = \frac{-k2\pi L}{Q}\left[T_2 - T_1\right].$$

$$\ln r_2 - \ln r_1 = \frac{K2\pi L}{Q}\left[T_1 - T_2\right] \longrightarrow (4)$$

Rate of heat flow through thick walled cylinder

$$Q = \frac{K2\pi L(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} \longrightarrow ⑤.$$

$$Q = \frac{K 2\pi L (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}.$$

Equation 5 can be written in terms of resistance as

$$Q = \frac{(T_1 - T_2)}{R} \longrightarrow (6)$$

where $R = \dfrac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K L}$.