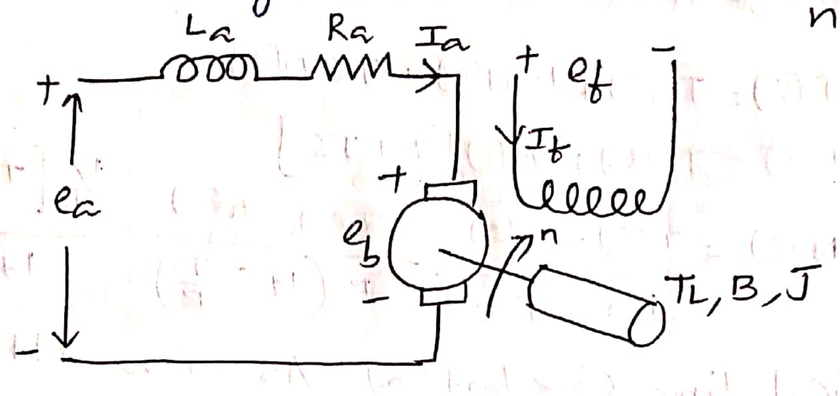
1. Transfer Function of DC Motor & Load:

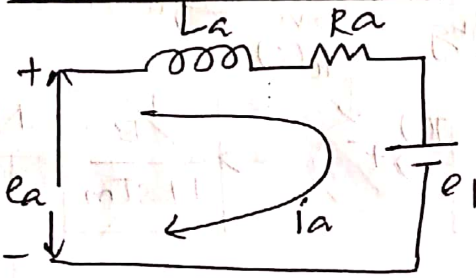
In Separately Excited DC Motor with Armature voltage control method, Field current is Constant but armature voltage is varied.

neglect rated speed



e_a - Armature voltage
R_a - Armature Resistance
e_b - back emf
e_f - field voltage
T - Motor torque
J - Moment of Inertia
L_a - Armature Inductance
I_a - Armature Current
I_f - Field Current
n - motor Speed
B - damping Co-efficient
T_L - Torque Load

Electrical Analysis:



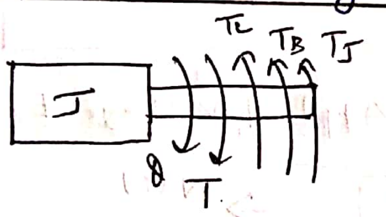
Applying KVL:

$$e_a = e_b + R_a i_a + L_a \frac{di_a}{dt}$$

$$e_b \propto \phi \frac{d\theta}{dt} \quad \therefore \frac{d\theta}{dt} = n \quad \text{——— (1)}$$

$$e_b = K_a \phi n \quad \text{——— (2)}$$

Mechanical Analysis:



Applying Newton's law:

$$T = T_L + T_B + T_J$$

$$T = T_L + B\frac{d\theta}{dt} + J\frac{d^2\theta}{dt^2}$$

$$\therefore T = T_L + Bn + J\frac{dn}{dt} \quad \text{——— (3)}$$

$$T = K_a \phi I_a \quad \text{——— (4)}$$

Taking Laplace Transform from eq (1) to (4)

$$E_a(s) = E_b(s) + R_a I_a(s) + sL_a I_a(s) \quad \text{——— (5)}$$

$$E_b(s) = K_a \phi N(s) \quad \text{——— (6)}$$

$$T(s) = T_L(s) + BN(s) + JsN(s) \quad \text{——— (7)}$$

$$T(s) = K_a \phi I_a(s) \quad \text{——— (8)}$$

From equation (5)

$$E_a(s) = E_b(s) + R_a I_a(s) + sL_a I_a(s)$$

$$E_a(s) = E_b(s) + I_a(s)\left[R_a + sL_a\right]$$

$$I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + sL_a} = \frac{E_a(s) - E_b(s)}{R_a\left(1 + s\frac{L_a}{R_a}\right)} = \frac{\frac{1}{R_a}\left[E_a(s) - E_b(s)\right]}{1 + s\tau_a}$$

$\tau_a = \frac{L_a}{R_a}$ electrical time Constant of the Armature ckt.

From equation ⑦

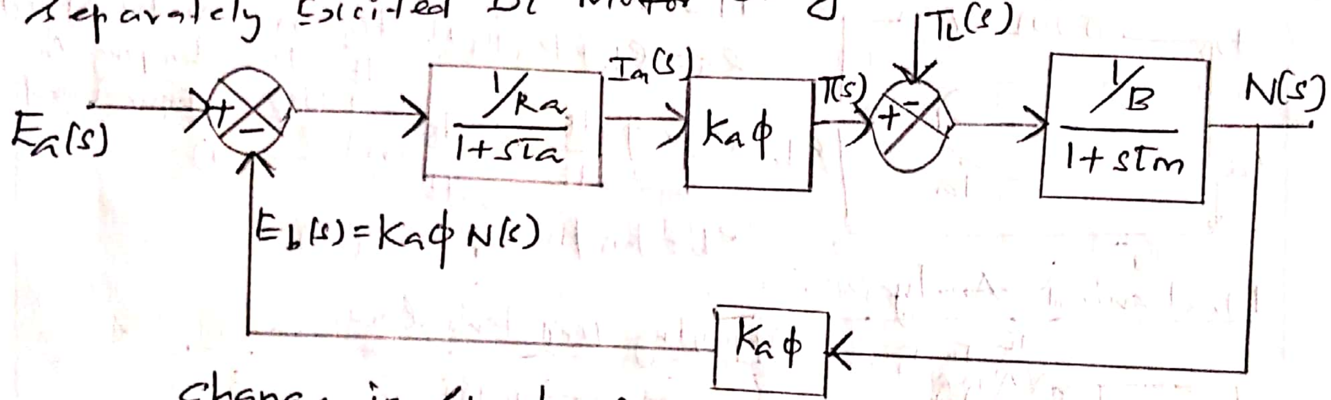
$$T(s) = T_L(s) + B\,N(s) + Js\,N(s)$$

$$T(s) = T_L(s) + N(s)\left[B + Js\right]$$

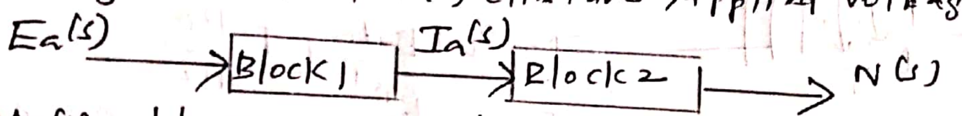
$$N(s) = \frac{T(s) - T_L(s)}{B + sJ} = \frac{T(s) - T_L(s)}{B\left(1 + s\frac{J}{B}\right)} = \frac{\frac{1}{B}\left[T(s) - T_L(s)\right]}{1 + s\tau_m}$$

$\tau_m = \frac{J}{B}$ mechanical time Constant of Armature ckt.

To form feedback loop in the form of Back Emf is separately Excited DC Motor using above equations.



change in speed N(s) disturb Applied Voltage $E_a(s)$ & $T_L(s)$



N(s) = block 1 Transfer function + block 2 transfer function.

$$\text{Block 1} = \frac{G_1(s)}{1 + G_1(s)H_1(s)} \cdot E_a(s), \quad \text{Block 2} = \frac{G_2(s)}{1 + G_2(s)H_2(s)} \cdot T_L(s)$$

So
$$N(s) = \frac{G_1(s)}{1 + G_1(s)H_1(s)}E_a(s) + \frac{G_2(s)}{1 + G_2(s)H_2(s)} \cdot T_L(s)$$

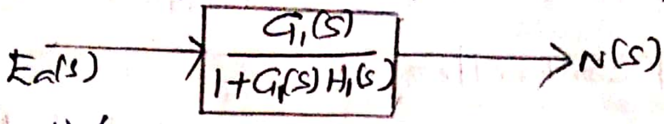
Where;

$$G_1(s) = \frac{1/R_a}{1 + s\tau_a}(K_a\phi)\frac{\frac{1}{B}}{1 + s\tau_m}$$

$$H_1(s) = K_a\phi$$

$$G_2(s) = \frac{-\left(1/B\right)}{1 + s\tau_m}, \quad H_2(s) = \frac{-(K_a\phi)^2/R_a}{1 + s\tau_a}$$

Now neglecting the load torque T_L,



$$\frac{N(s)}{E_a(s)} = \frac{\dfrac{(K_a\phi)/R_aB}{(1+s\tau_a)(1+s\tau_m)}}{1+\dfrac{(K_a\phi)/R_aB}{(1+s\tau_a)(1+s\tau_m)}\cdot K_a\phi} = \frac{(K_a\phi)/R_aB}{(1+s\tau_a)(1+s\tau_m)+(K_a\phi)^2/R_aB}$$

$$\frac{N(s)}{E_a(s)} = \frac{K_a\phi}{R_aB(1+s\tau_a)(1+s\tau_m)+(K_a\phi)^2}, \quad \text{If } \tau_a \ll \tau_m, \ \tau_a \text{ neglected}$$

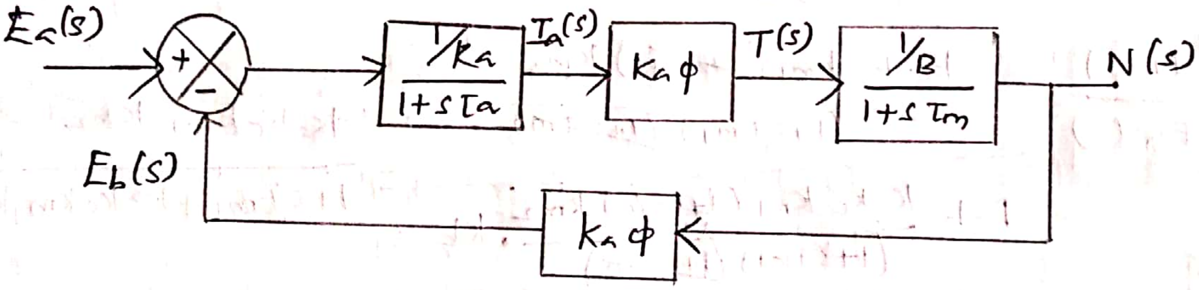
$$\frac{N(s)}{E_a(s)} = \frac{K_a\phi}{R_aB(1+s\tau_m)+(K_a\phi)^2} = \frac{K_a\phi}{R_aB+R_aBs\tau_m+(K_a\phi)^2}$$

Dividing $R_aB+(K_a\phi)^2$ in all terms

$$\frac{N(s)}{E_a(s)} = \frac{K_a\phi/R_aB+(K_a\phi)^2}{1+\dfrac{sR_aB}{R_aB+(K_a\phi)^2}\tau_m} = \frac{K_m}{1+s\tau_{m1}}, \quad \boxed{\frac{N(s)}{E_a(s)} = \frac{K_m}{1+s\tau_{m1}}}$$

$$\tau_{m1} = \frac{R_aB\tau_m}{R_aB+(K_a\phi)^2}, \quad K_m = \frac{K_a\phi}{(K_a\phi)^2+R_aB}$$

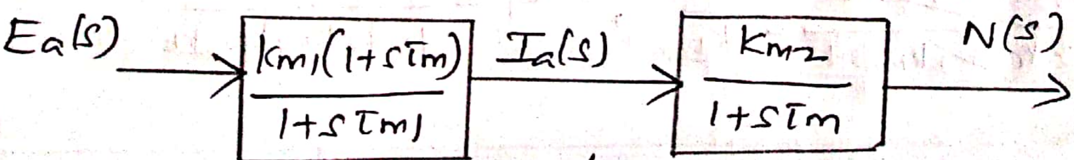
$\tau_{m1} \ll \tau_m$, neglecting T_L



$$\frac{N(s)}{I_a(s)} = \frac{1/B}{1+s\tau_m}\times K_a\phi = \frac{K_a\phi/B}{1+s\tau_m} = \frac{K_{m2}}{1+s\tau_m}, \quad K_{m2} = \frac{K_a\phi}{B}$$

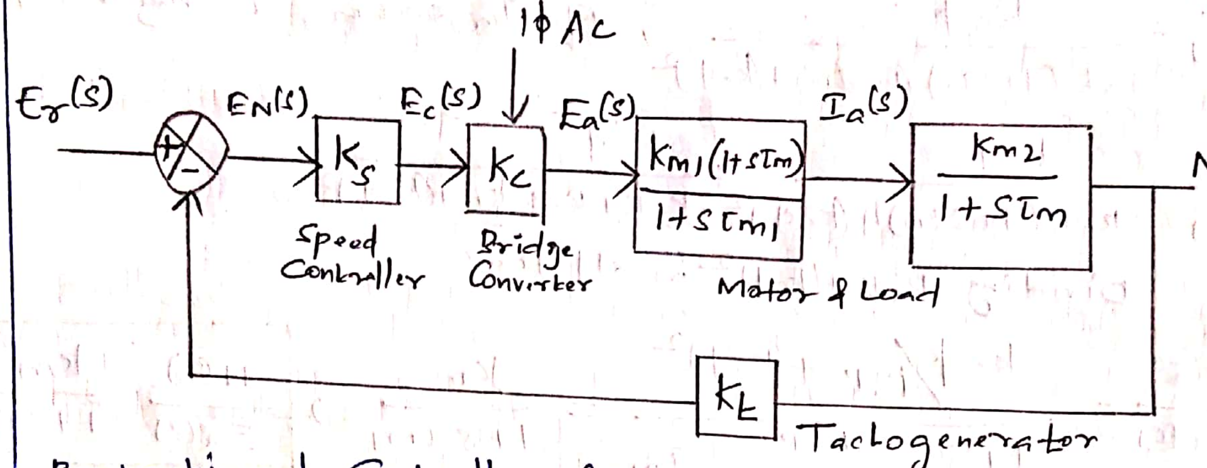
$$\frac{I_a(s)}{E_a(s)} = \frac{N(s)}{E_a(s)}\times\frac{I_a(s)}{N(s)} = \frac{K_m}{1+s\tau_{m1}}\times\frac{(1+s\tau_m)B}{K_a\phi} = \frac{K_mB(1+s\tau_m)}{K_a\phi(1+s\tau_{m1})}$$

$$\boxed{\frac{I_a(s)}{E_a(s)} = \frac{K_{m1}(1+s\tau_m)}{1+s\tau_{m1}}}, \quad K_{m1} = \frac{K_mB}{K_a\phi}, \quad K_m = K_{m1}\times K_{m2}$$



2. Design of Speed Controller:

If a dc tachogenerator is attached to Motor shaft, a speed signal fed back to back emf and Error EN(s) control the Armature voltage using speed controller and 1ϕ & 3ϕ converter(or) chopper.



Proportional Controller (P-Controller)

proportional Integral Controller (PI)

P- Controller:

$$G_1(s) = k_s k_c \frac{k_{m1}(1+sT_m)}{1+sT_{m1}} \times \frac{k_{m2}}{1+sT_m}$$

$$H(s) = K_t$$

$$\frac{N(s)}{E_r(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\frac{N(s)}{E_r(s)} = \frac{\dfrac{k_s k_c k_{m1} \cancel{(1+sT_m)} k_{m2}}{(1+sT_{m1})\cancel{(1+sT_m)}}}{1+\dfrac{k_s k_c k_{m1} \cancel{(1+sT_m)} k_{m2}}{(1+sT_{m1})\cancel{(1+sT_m)}} \cdot k_t} = \frac{k_s k_c k_{m1} k_{m2}}{1+sT_{m1}+k_s k_c k_{m1} k_{m2} k_t}$$

Dividing by $1+k_s k_c k_{m1} k_{m2} k_t$

$$\frac{N(s)}{E_r(s)} = \frac{\dfrac{k_s k_c k_{m1} k_{m2}}{1+k_s k_c k_{m1} k_{m2} k_t}}{1+s\dfrac{T_{m1}}{k_s k_c k_{m1} k_{m2} k_t}} = \frac{k_1}{1+sT_1}$$

$$K_1 = \frac{k_s k_c k_{m1} k_{m2}}{1+k_s k_c k_{m1} k_{m2} k_t},$$ $k_s k_c k_{m1} k_{m2} k_t >> 1$, neglect 1

$$K_1 = \frac{k_s k_c \cancel{k_{m1} k_{m2}}}{k_s k_c \cancel{k_{m1}} k_{m2} k_t} = \frac{1}{K_t}, \quad T_1 = \frac{T_{m1}}{k_s k_c k_{m1} k_{m2} k_t}$$

$$\frac{I_a(s)}{N(s)} = \frac{1+s\tau_{m1}}{K_{m2}}$$

$$\frac{I_a(s)}{E_r(s)} = \frac{N(s)}{E_r(s)} \times \frac{I_a(s)}{N(s)} = \frac{K_1}{1+s\tau_1} \times \frac{1+s\tau_m}{K_{m2}} = \frac{K_1(1+s\tau_m)}{K_{m2}(1+s\tau_1)}$$

$$\therefore I_a(s) = \frac{K_1(1+s\tau_m)}{K_{m2}(1+s\tau_1)} \cdot E_r(s) \quad , \quad E_r(s)\ \text{Step Input} = \frac{E_r}{s}$$

$$I_a(s) = \frac{K_1(1+s\tau_m)}{K_{m2}(1+s\tau_1)} \cdot \frac{E_r}{s} = \frac{K_1 E_r(1+s\tau_m)}{K_{m2}s(1+s\tau_1)} = \frac{K_1 E_r(1+s\tau_m)}{K_{m2}s\tau_1\left(\frac{1}{\tau_1}+s\right)}$$

$\div K_{m2}\tau_1$ both numerator & denominator

using partial fraction

$$I_a(s) = \frac{\frac{K_1 E_r}{K_{m2}\tau_1}(1+s\tau_m)}{\frac{\cancel{K_{m2}} s \cancel{\tau_1}}{\cancel{K_{m2}\tau_1}}\left(s+\frac{1}{\tau_1}\right)} = \frac{A_1}{s} + \frac{A_2}{s+\frac{1}{\tau_1}} = \frac{A_1(s+1/\tau_1)+A_2 s}{s(s+1/\tau_1)}$$

$$\frac{K_1 E_r}{K_{m2}\tau_1}(1+s\tau_m) = A_1(s+1/\tau_1) + A_2 s$$

$$\frac{K_1 E_r}{K_{m2}\tau_1} + \frac{sK_1 E_r \tau_m}{K_{m2}\tau_1} = A_1 s + \frac{A_1}{\tau_1} + A_2 s = (A_1+A_2)s + \frac{A_1}{\tau_1}$$

Equate Constant term $\rightarrow$ $\frac{K_1 E_r}{K_{m2}\tau_1} = \frac{A_1}{\tau_1}$, $\therefore A_1 = \frac{K_1 E_r \cancel{\tau_1}}{K_{m2}\cancel{\tau_1}} = \frac{K_1 E_r}{K_{m2}}$

Equate 's' term $\rightarrow$ $\therefore \frac{K_1 E_r \tau_m}{K_{m2}\tau_1} = A_1 + A_2 = \frac{K_1 E_r}{K_{m2}} + A_2$

$$A_2 = \frac{K_1 E_r}{K_{m2}}\left(\frac{\tau_m}{\tau_1} - 1\right)$$

$$I_a(s) = \frac{K_1 E_r}{K_{m2} s} + \left[\frac{K_1 E_r}{K_{m2}}\left(\frac{\tau_m}{\tau_1}-1\right)\right] \Big/ (s+1/\tau_1)$$

$$= \frac{K_1 E_r}{K_{m2}}\left[\frac{1}{s} + \frac{\tau_m - \tau_1}{\tau_1} \Big/ (s+1/\tau_1)\right]$$ $\rightarrow$ Taking Inverse Laplace

$$\therefore I_a(t) = \frac{K_1 E_r}{K_{m2}}\left[1 + \frac{\tau_m - \tau_1}{\tau_1} \cdot e^{-t/\tau_1}\right]$$

$$\therefore I_a(\infty) = \frac{K_1 E_r}{K_{m2}}\left[1 + \frac{\tau_m - \tau_1}{\tau_1} e^{-\infty}\right] = \frac{K_1 E_r}{K_{m2}}$$

$$\frac{I_a(t)}{I_a(\infty)} = \frac{\frac{K_1 E_r}{K_{m2}}\left[1+\frac{(\tau_m-\tau_1)}{\tau_1} \cdot e^{-t/\tau}\right]}{E_r K_1 / K_{m2}} = \boxed{1+\left(\frac{\tau_m-\tau_1}{\tau_1}\right)e^{-t/\tau}}$$

3. Armature voltage Control & Field Weakening Mode with closed loop Speed Control of DC Drive

In closed loop block diagram of speed Control (i) Below the Base Speed (ii) Above the Base Speed.

Loop Consist of Inner current loop.
↳ outer speed loop.

Below the Base Speed → Constant Field current
Variable Armature voltage

Above the Base Speed → constant Armature voltage
Variable Field current

Fully Controlled Converter Forward Braking is not possible

PI Controller is mostly used good steady state Accuracy & Filter out Noise

Below the Base Speed:

Eref (0.85 to 0.95) of Rated Armature voltage (Eref) Compared with (E_b) → Err → At rated operation
→ αf is Zero, ↑ reference speed → Current limiter saturates & set maximum value of permissible current.

If drive accelerating to set point → Speed ↑ with Eref current limiter desaturates → $T_m = T_L$.

If Speed ↓ref, Iref → 0, drive deaccelerate to Load torque.

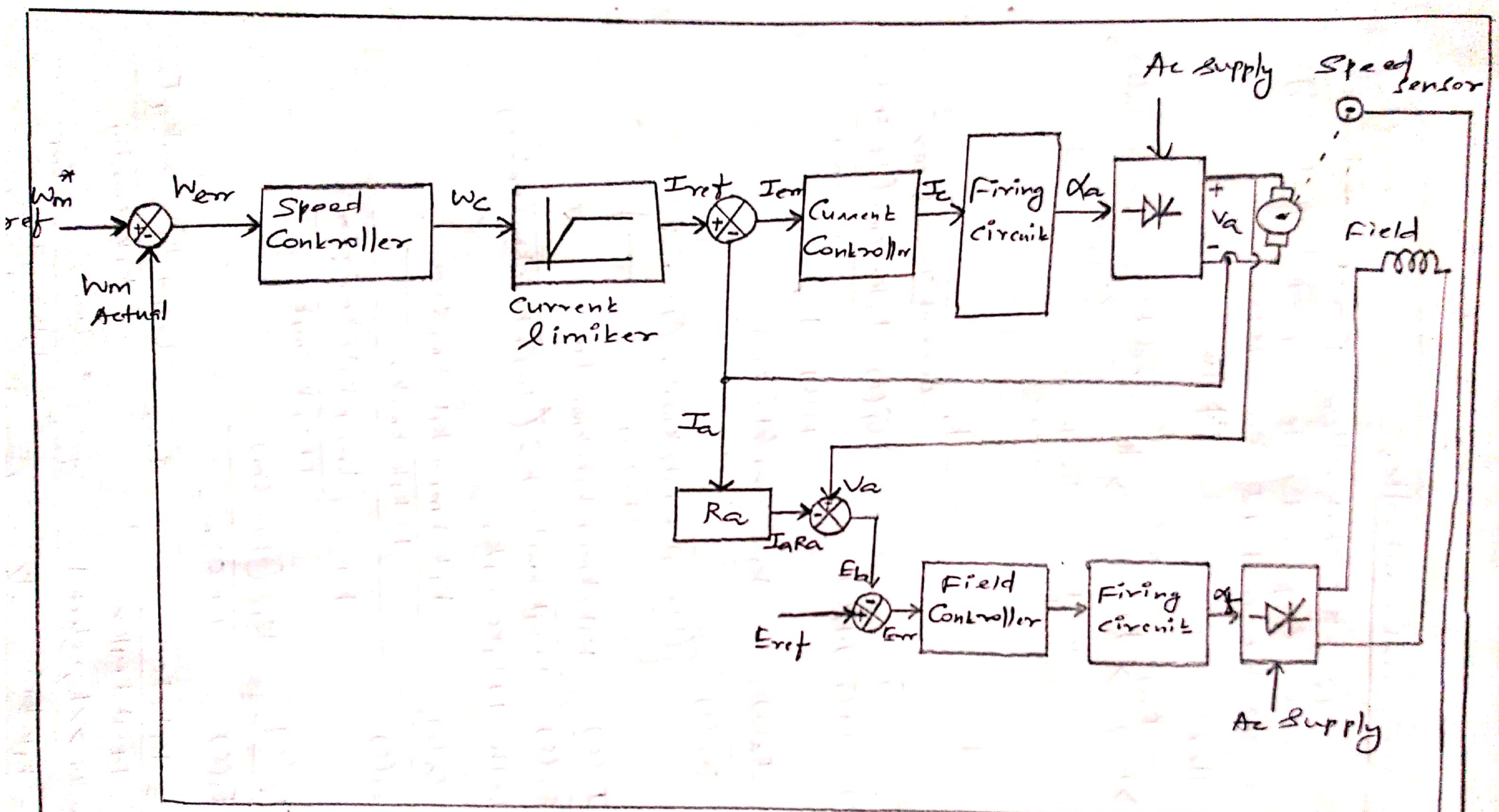
Above the Base Speed:

When speed of the Motor is close to base speed, the Field Controller Comes out of saturation reference speed set point choosen to be Higher than base speed.

Current limiter sets the Corresponding maximum permissible value of reference Current, $\alpha_A \downarrow$, $V_a \uparrow$ the Motors starts accelerating & ↑ E_b.

When (V_a) small Werr small → High value of Field time Constant

Maintaining V_a Constant & Varying Field, speed of the Motor above base speed.

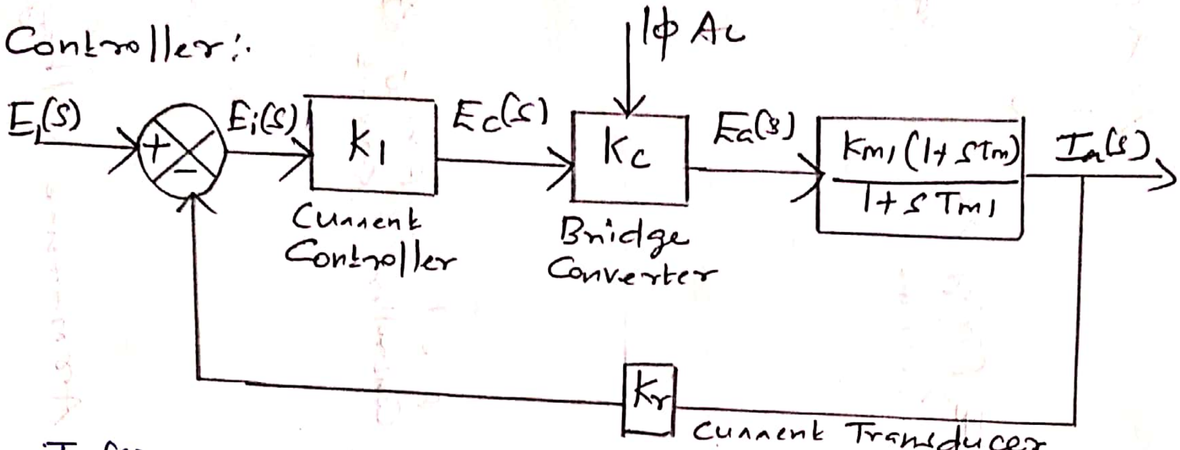


Closed loop Speed Control of Armature Voltage & Field Weakening

4. Design of Current Controller:

In this controller first construct inner current loop and outer speed loop. Using both P controller and PI controller.

P-Controller:



$$\frac{I_a(s)}{E_r(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}, \qquad G(s) = \frac{K_1 K_c K_{m1}(1+sT_m)}{1+sT_{m1}}$$

$$H(s) = K_r$$

$$\frac{I_a(s)}{E_r(s)} = \frac{\dfrac{K_1 K_c K_{m1}(1+sT_m)}{1+sT_{m1}}}{\left[\dfrac{(1+sT_{m1}) + K_1 K_c K_{m1} K_r (1+sT_m)}{1+sT_{m1}}\right]}$$

$$\frac{I_a(s)}{E_r(s)} = \frac{K_1 K_c K_{m1} + K_1 K_c K_{m1} s T_m}{1+K_1 K_c K_{m1} K_r + s\left(T_{m1} + K_1 K_c K_{m1} K_r T_m\right)}$$

$\div$ both $1+K_1 K_c K_{m1} K_r$

$$\frac{I_a(s)}{E_r(s)} = \frac{\dfrac{K_1 K_c K_{m1} + K_1 K_c K_{m1} s T_m}{1+K_1 K_c K_{m1} K_r}}{1 + s\left(\dfrac{T_{m1} + K_1 K_c K_{m1} K_r T_m}{1+K_1 K_c K_{m1} K_r}\right)}$$

$$\frac{I_a(s)}{E_r(s)} = \frac{K_{1c}(1+sT_m)}{1+sT_{m2}}$$

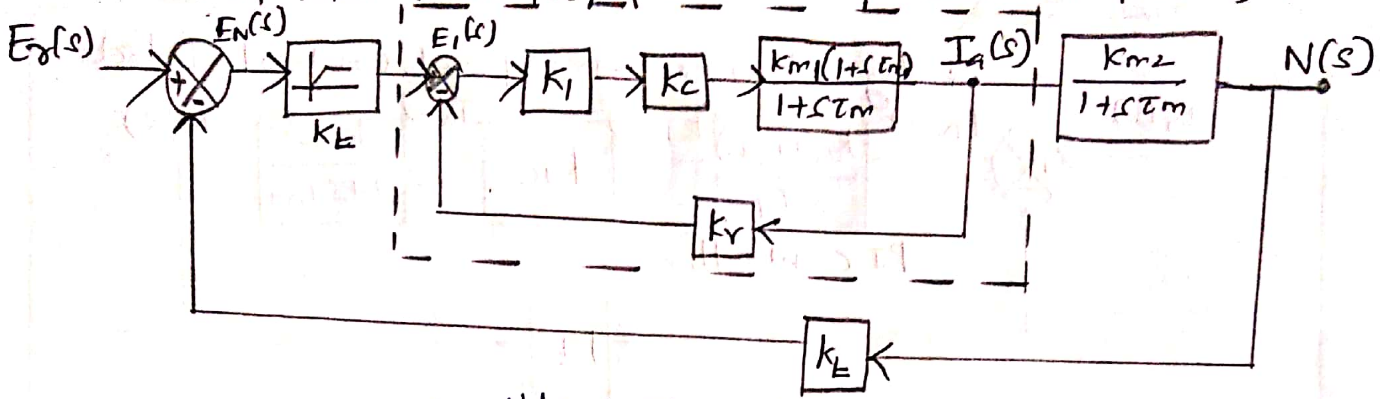
$\therefore K_{1c} = \dfrac{K_1 K_c K_{m1}}{1+K_1 K_c K_{m1} K_r}$, $\qquad T_{m2} = \dfrac{T_{m1} + K_1 K_c K_{m1} K_r T_m}{K_1 K_c K_{m1} T_m}$

$K_1 K_c K_{m1} K_r >> 1$, neglect 1 $\qquad T_{m2} = \dfrac{T_{m1}}{\cancel{K_1 K_c K_{m1} T_m}} + T_m$

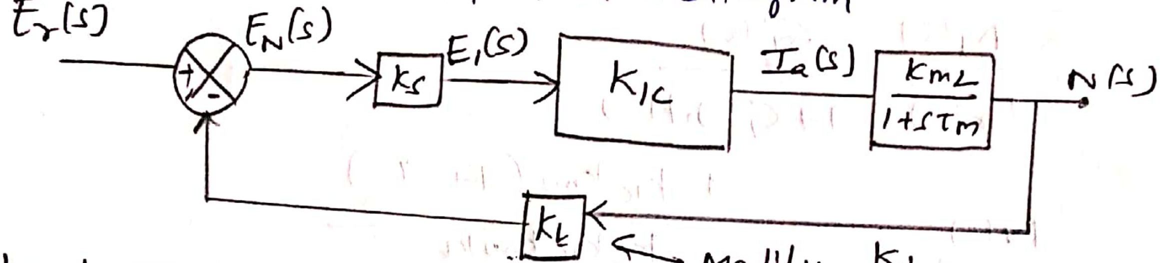
$K_{1c} = \dfrac{K_1 K_c \cancel{K_{m1}}}{\cancel{K_1 K_c K_{m1}} K_r} \approx 1/K_r$ $\qquad T_m >> T_{m1}$, neglect T_{m1}

$$\frac{I_a(s)}{E_1(s)} = \frac{k_{1c}(1+sT_m)}{1+sT_{m2}} = \frac{1+sT_m}{k_r(1+sT_{m2})} = \frac{1}{k_r} = k_{1c} \quad \therefore k_{1c} = \frac{1}{k_r}$$

Now Add with Inner loop & outer loop (Current & speed)



Modified Blockdiagram



Modify $\frac{K_1}{1+sT_t}$

Without Tachogenerator:-

$$\frac{N(s)}{E_r(s)} = \frac{K_2}{1+sT_2} \quad \therefore k_2 = \frac{k_s k_{1c} k_{m2}}{1+k_s k_{1c} k_{m2} k_t}, \quad k_2 = 1/k_t$$

$k_s k_{1c} k_{m2} k_t >> 1$, neglect 1

$$\therefore \frac{I_a(s)}{E_r(s)} = \frac{N(s)}{E_r(s)} \times \frac{I_a(s)}{N(s)} \qquad T_2 = \frac{T_m}{k_s k_{1c} K_{m2} k_t}$$

$$= \frac{k_2}{1+sT_2} \times \frac{1+sT_m}{k_{m2}} = \frac{k_2(1+sT_m)}{k_{m2}(1+sT_2)}$$

$$\therefore \frac{N(s)}{I_a(s)} = \frac{k_{m2}}{1+sT_m}, \quad N(s) = \frac{k_{m2}}{1+sT_m} \cdot \frac{I_a}{s}$$

Tachogenerator with Filter:-

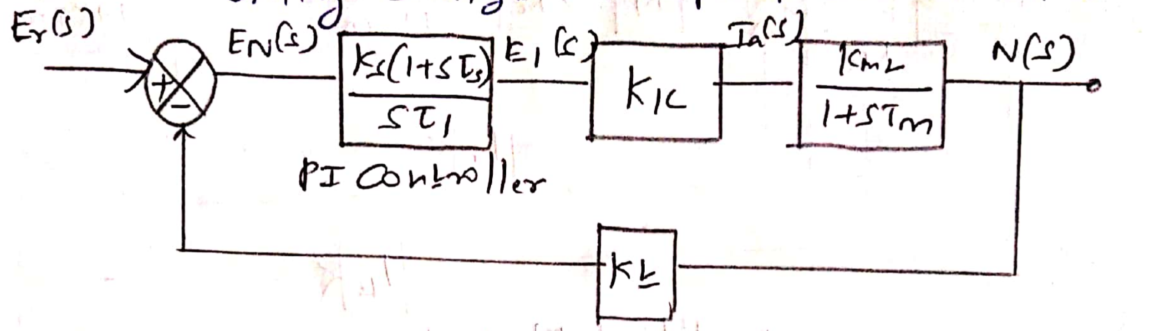
$$\frac{N(s)}{E_r(s)} = \frac{k_s k_c k_{m2} k_{1c}(1+sT_t)}{\left(1+\frac{s(T_t+T_m)}{k_1}+\frac{s^2 T_m T_t}{k_1}\right)(1+k_t k_s k_{m2} k_{1c})}$$

$$\frac{I_a(s)}{E_r(s)} = \frac{N(s)}{E_r(s)} \times \frac{I_a(s)}{N(s)}, \quad \frac{1+sT_m}{k_{m2}} = \frac{I_a(s)}{N(s)}$$

$$\frac{I_a(s)}{E_r(s)} = \frac{k_s k_{1c}(1+sT_t)(1+sT_m)}{(1+k_t k_s k_{m2} k_{1c})\left[1+\frac{s(T_t+T_m)}{k_1}+\frac{s^2 T_m T_t}{k_1}\right]}$$

PI Controller:-

only change in Speed Control Block.



$$\frac{N(s)}{E_r(s)} = \frac{G(s)}{1+G(s).H(s)}$$

$$\frac{N(s)}{E_r(s)} = \frac{\dfrac{K_s K_{1c} K_{m2}(1+sT_s)}{K_s K_{1c} K_{m2} K_t}}{1+\dfrac{sT_s(1+K_s K_{1c} K_{m2} K_t)}{K_s K_{1c} K_{m2} K_t}+\dfrac{s^2 T_s T_m}{K_s K_{1c} K_{m2} K_t}}$$

$$\frac{N(s)}{E_r(s)} = \frac{\dfrac{1}{K_t}(1+sT_s)}{1+sT_s+s^2T_sT_2} = \frac{1}{K_t}\,\frac{1+sT_s}{1+sT_s+s^2T_sT_2}$$

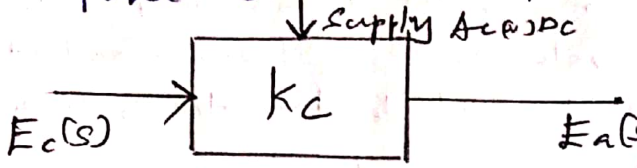
$$\frac{I_a(s)}{E_r(s)} = \frac{N(s)}{E_r(s)} \times \frac{I_a(s)}{N(s)}, \quad \frac{I_a(s)}{N(s)} = \frac{1+sT_m}{K_{m2}}$$

$$\frac{I_a(s)}{E_r(s)} = \frac{1}{K_t}\,\frac{1+sT_s}{1+sT_s+s^2T_sT_2} \times \frac{1+sT_m}{K_{m2}}$$

$$\therefore \boxed{\frac{I_a(s)}{E_r(s)} = \left(\frac{1}{K_t K_{m2}}\right)\frac{(1+sT_s)(1+sT_m)}{(1+sT_s+s^2T_sT_2)}}$$

5. Transfer Function of power Converter:-

Power Converter Gain - K_c



Converter Transfer function $G_c(s) = \frac{E_a(s)}{E_c(s)}$

$$G_c(s) = \frac{K_c}{1+sT_c}$$

K_c - Converter gain
T_c - Converter timedelay
s - laplace operator

Converter gain for maximum Control voltage E_{cm}

(1φ)

$$K_c = \frac{2V_m}{\pi E_{cm}} = \frac{2\sqrt{2}V_s}{\pi E_{cm}}$$

$$K_c = 0.9\frac{V_s}{E_{cm}}$$

Where
V_s - 1φ Ac voltage
E_{cm} - maximum Control voltage

(3φ)

$$K_c = \frac{3V_{ml}}{\pi E_{cm}} = \frac{3\sqrt{2}V_L}{\pi E_{cm}}$$

$$K_c = \frac{1.35 V_L}{E_{cm}}$$

Delay may be treated one half of this Interval

$$T_c = \frac{60/2}{360} \times \text{time period of one cycle}$$

$$T_c = \frac{1}{12} \times \frac{1}{f_s}$$

Converter when Modeled with its gain & time delay

$$G_c(s) = K_c e^{-sT_c}$$

$$\therefore \boxed{G_c(s) = \frac{K_c}{1+sT_c}}$$

6. Converter Selection & characteristics:-

Depending upon the specifications of Motor and Load, the ratings of power converter choosen.

I_{max} – motor current is assumed continuous & Rms value of each device conduct 120° electrical. It's not suitable for discontinuous conduction.

RMS value of current in power device:-

$$I_{rms} = \frac{I_{max}}{\sqrt{3}} = 0.5771\, I_{max}$$

Voltage rating of device:-

$$V_{rg} = \sqrt{2}V \quad (\text{line to line voltage})$$

Rms value of fundamental component of supply current I_1:-

$$I_1 = \frac{2\sqrt{3}}{\sqrt{2}\pi} I_{max} = \frac{\sqrt{2}\sqrt{3}}{\pi} I_{max} = 0.78\, I_{max}$$

power rating of converter:-

$$P_o = V_o I_{max} = (1.35 V \cos\alpha) I_{max}$$

$$P_o = 1.35 V I_{max} \cos\alpha$$

Assume No loss in Converter:- $\therefore$ α – PF angle

$$P_{in} = P_o = \sqrt{3} V I_1 \cos\alpha \quad \text{———} \quad (1)$$

$\therefore$ Reactive power $Q_i = 1.35 V I_{max} \sin\alpha$ ——— (2)

Apparent power $S = \sqrt{(P_i)^2 + (Q_i)^2}$

$$\boxed{S = 1.35 V I_{max} = \sqrt{3} V I_1}$$