Ex: A 3φ, 4 pole brushless PM motor has 36 stator slots, Each phase winding
① is made up of 3 coils per pole with 20 turns per coil. The coil span is seven slots. If the fundamental component of magnet flux is 2 mwb. Calculate the open circuit phase emf at 3000 rpm.

Soln:-

Given data, 3φ, $P = 4$ brushless PM motor

No. of stator slots = 36

Each phase winding coils/pole = 3 ; turns/coil = 20

Coil span = 7 slots , flux/pole $\phi = 2 \times 10^{-3}$ wb

$N = 3000$ rpm

To find

open circuit phase emf E_{ph}

frequency $f = \frac{PN}{120} = \frac{4 \times 3000}{120} = 100$ Hz

No. of turns/phase $T_{ph} = \frac{\text{No. of turns}}{\text{coils}} \times \text{coils/pole} \times \text{no. of poles} \times \frac{1}{\text{phase}}$

$= 20 \times 3 \times 4 = 240$

distribution factor $k_{d1} = \frac{\sin m\beta/2}{m \sin \beta/2}$

m = slots/pole/ph β = slot angle = $180°/n = 180/9 = 20°$

n = slots/pole = $36/4 = 9$

$m = 9/3 = 3$ so $k_{d1} = \frac{\sin\left(\frac{3 \times 20°}{2}\right)}{3 \sin \frac{20°}{2}} = 0.9598$

Full pitch = 9 slots/pole

Given that coil span = 7 slots

So short pitched by 2 slots ⇒ $2 \times 20° = 40°$

pitch factor $k_{p1} = \cos\frac{\alpha}{2} = \cos 40°/2 = 0.9396$

where $\alpha \rightarrow$ Angle of short pitch

Slewing factor $k_{s1} = \frac{\sin \sigma/2}{\sigma/2} = 1$ (Assume)

winding factor $k_{w1} = k_{s1} k_{p1} k_{d1} = 1 \times 0.9396 \times 0.9598$

$= 0.9018$

open circuit phase emf $E_{ph} = 4.44 f \phi_m k_{w1} T_{ph}$

$$\boxed{E_{ph} = 192.19\ V}$$

Ex: compare permanent Magnet brushless DC motor with permanent magnet synchronous motor based on their performance parameters.

Soln:-

PM brushless DC motor	PM Synchronous motor
① Flux per pole $\phi = B_g \tau l$ wb	$\phi = B_{av} \tau l$ wb
② line currents to the motor $I_{rms} = \sqrt{\frac{2}{3}} I_d$	$I_{rms} = \frac{I_m}{\sqrt{2}}$
③ RMS value of the line current $I_{rms} = \frac{1}{\sqrt{3}} I_d$	$I_{rms} = \frac{I_m}{2}$
④ peak value of the device current $I_p = I_d$	$I_p = I_m$
⑤ VA rating of the device $(VA)_{rms} = V_{dc} \frac{I_d}{\sqrt{3}}$	$(VA)_{rms} = V_{dc} \frac{I_m}{\sqrt{2}}$

(6) peak value of the device voltage

$$V_p = V_{dc}$$

$$V_p = V_{dc}$$

(7) output VA of the switching circuitry

$$\text{output} = V I_d$$

$$\text{output} = \frac{\sqrt{3}}{2} V I_m$$

(8) Torque developed by the motor

$$T = 4 B_g r l I_d T_{ph}$$

$$T = \pi B_{gm} A_m r l \sin\beta$$

(9) RMS VA rating of the switching circuitry

$$(VA)_{rms} = 6\left[\frac{V_d I_d}{\sqrt{3}}\right]$$

$$(VA)_{rms} = 6\left[\frac{V_{dc} I_{rms}}{2}\right]$$

(10) Torque per I_{rms}

$$T = 4 B_g r l T_{ph} I_d$$

$$= 4\left(\frac{\phi}{\tau l}\right) r l T_{ph}\left(\frac{\sqrt{3}}{2} I_{rms}\right)$$

$$\frac{T}{I_{rms}} = 4\sqrt{3/2}\ \frac{\phi}{\tau}\ l\, T_{ph}$$

$$T_m = \frac{3}{2}\cdot \pi/2\ I_m \hat{B}\, r l\, T_{ph}$$

$$= 3/2 \cdot \pi/2\ \sqrt{2} I\left(\frac{\pi}{2}\frac{\phi}{\tau l}\right) r l\, T_{ph}$$

(3)

$$\frac{T_m}{I_{rms}} = \frac{3\pi^2\sqrt{2}}{8}\cdot \frac{\phi_r T_{ph}}{\tau}$$

$$\text{Comparing } \frac{T/I_{rms}}{T/I_{rms}} = \frac{4\sqrt{3}/\sqrt{2}}{3\pi^2\sqrt{2}/8} = 0.93$$

(11) Torque per I_{peak}

$$T = 4 B_g r l T_{ph} I_d$$

$$\phi = B_g \tau l$$

$$T = 4\left(\frac{\phi}{\tau l}\right) r l\, T_{ph} I_d$$

$$\frac{T}{I_d} = \frac{4\phi}{\tau} r T_{ph}$$

$$T_m = \frac{3}{2}\cdot \pi/2\ I_m \hat{B}\, r l\, T_{ph}$$

$$\phi = \frac{2}{\pi} \hat{B} \tau l$$

$$T_m = \frac{3}{2}\cdot\frac{\pi}{2} I_m\left(\frac{\pi}{2}\frac{\phi}{\tau l}\right) r l\, T_{ph}$$

$$\frac{T_m}{I_m} = \frac{3\pi^2}{8}\frac{\phi}{\tau} r\, T_{ph}$$

Comparing $\dfrac{T/I_d}{T_m/I_m} = \dfrac{\dfrac{4\phi}{\tau} r\, T_{ph}}{\dfrac{3\pi^2}{8}\dfrac{\phi}{\tau} r\, T_{ph}} = \dfrac{3\times 2}{3\pi^2} = 1.08$

(12) Torque per RMS ampere

$$T = 4 B_g r l\, T_{ph}\, I_d$$

$$I_{rms} = \sqrt{2/3}\, I_d$$

$$\frac{T}{I_{rms}} = \frac{4 B_g r l\, T_{ph}\, I_d}{\sqrt{2/3}\, I_d}$$

$$= 2\sqrt{6}\, B_g r l\, T_{ph}$$

$$T_m = \frac{3}{2}\,\frac{\pi}{2}\, I_m \hat{B} r l\, T_{ph}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\frac{T_m}{I_{rms}} = \frac{\frac{3}{2}\cdot\frac{\pi}{2}\, I_m \hat{B} r l\, T_{ph}}{I_m/\sqrt{2}}$$

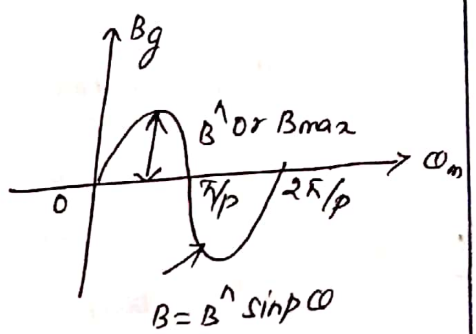
$$= \frac{3}{2\sqrt{2}} \hat{B} r l\, T_{ph}$$

Comparing $\dfrac{T/I_{rms}}{T_m/I_{rms}} = \dfrac{2\sqrt{6}\, B_g r l\, T_{ph}}{\dfrac{3}{2\sqrt{2}} \hat{B} r l\, T_{ph}} = 1.47$

(3) Ex: Derive the emf equation of ideal and practical PMSM.

Soln:-

The flux density $B = \hat{B} \sin p\omega$ at the strip



Incremental flux in the strip $d\phi = B \times Area$

$$d\phi = \hat{B} \sin p\omega \times l r\, d\omega$$

$$d\phi = \hat{B}\, l r \sin p\omega\, d\omega$$

where $l \rightarrow$ length of the armature in m

$r \rightarrow$ radius of the armature in m

The flux enclosed by the coil $\phi = \int_{\omega_m t}^{\omega_m t + \pi/p} \hat{B}\, l r \sin p\theta \, d\theta$

$$\phi = \hat{B} l r \left[-\frac{\cos p\theta}{p} \right]_{\omega_m t}^{\omega_m t + \pi/p}$$

$$= \frac{\hat{B} l r}{p} \left[-\cos(p\omega_m t + \pi) + \cos p\omega_m t \right]$$

$$= \frac{\hat{B} l r}{p} \, 2 \cos p\omega_m t$$

According to the faraday's law of electromagnetic induction, emf induced in the single turn coil is given by,

$$e = -N \frac{d\phi}{dt} = -\frac{d\phi}{dt} \quad \text{since } N = 1$$

$$e = -\frac{d}{dt}\left[\frac{2\hat{B} l r}{p} \cos p\omega_m t \right] = \frac{2\hat{B} l r \, p\omega_m \sin p\omega_m t}{p}$$

$$e = 2\hat{B} l r \, \omega_m \sin p\omega_m t$$

Let $T_{ph} \rightarrow$ no. of turns / phase

max value of Emf induced / phase $E_{ph} = 2\hat{B} l r \, \omega_m T_{ph} \sin p\omega_m t$

where $\hat{E}_{ph} = 2\hat{B} r l T_{ph} \omega_m$ and $p\omega_m = \omega_e$

$$E_{ph} = \hat{E}_{ph} \sin \omega_e t$$

RMS value of Emf induced / ph $E_{RMS\,ph} = \frac{\hat{E}_{ph}}{\sqrt{2}} = \sqrt{2}\hat{B} r l T_{ph} \omega_m$

where $\omega_m = \frac{\omega_e}{p}$

$$\phi = B_{av} \tau l = B_{av} \cdot \frac{2\pi r}{2p} \times l$$

avg value of flux density $= 2/\pi \, \hat{B} \Rightarrow \phi_m = \frac{2}{\pi} \hat{B} \frac{\pi r}{p} l$

$$\phi_m = \frac{2\hat{B} r l}{P}$$

$$\hat{B} r l = \frac{P\phi_m}{2}$$

$$E_{ph} = \sqrt{2}\, \hat{B} r l\, \omega_m T_{ph} \quad \text{volts}$$

$$E_{ph} = \sqrt{2}\left(\frac{P\phi_m}{2}\right)\omega_m T_{ph} = \sqrt{2}\, \frac{P\phi_m}{2}\, \frac{\omega_e}{P}\, T_{ph}$$

$$= \sqrt{2}\, \frac{P\phi_m}{2} \cdot \frac{2\pi f}{P}\, T_{ph} = \sqrt{2}\, \pi f \phi_m T_{ph}$$

$E_{ph} = 4.44 f \phi_m T_{ph}$ volts → rms value of the induced emf per phase of the armature winding of an ideal BLPM sine wave motor.

In a practical BLPM sine wave motor, special care has been taken to design

So winding factor $K_{w1} = K_{s1} K_{p1} K_{d1}$

where K_{s1} = Skewing factor = $\frac{\sin\frac{\sigma}{2}}{\sigma/2} \simeq 1$

$\sigma \rightarrow$ skew angle (rad)

K_{p1} = pitch factor = $\cos\frac{\alpha}{2} \simeq 1$ (Assume full pitch)

$\alpha \rightarrow$ angle of short pitch

K_{d1} = distribution factor = $\frac{\sin m\beta/2}{m \sin\beta/2} \simeq 1$ (Assume concentrated winding)

β = slot angle = $180°/n$

n = slots/pole

m = slots/pole/phase

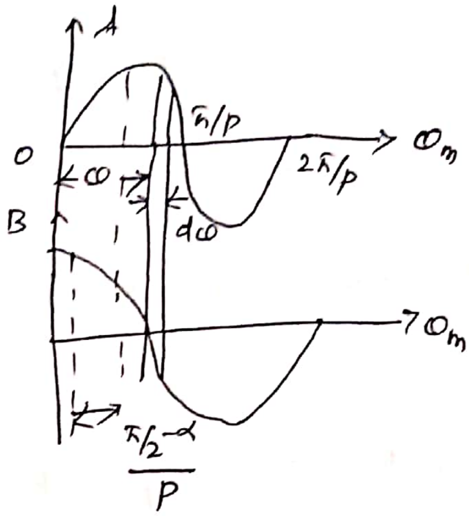
RMS value of the induced emf/ph $E_{ph} = 4.44 f \phi_m K_{w1} T_{ph}$ volts.

Ex: Derive the torque equation of an ideal and practical PMSM.

Soln: 4

Let the ampere conductor distribution of ideal BLPM sinewave motor is

$A = \hat{A} \sin p\theta$



Let us assume that the axis of armature ampere conductor distribution be displaced from the axis of the flux density distribution by an angle $(\pi/2 - \alpha)$

$B = \hat{B} \cos(p\theta - \alpha)$

Force experienced by the armature conductors in the strip $d\theta$ $d\vec{F} = B \, l \, A \, d\theta$

$d\vec{F} = \hat{B} \cos(p\theta - \alpha) \, l \, \hat{A} \sin p\theta \, d\theta$

$= \hat{A}\hat{B} l \sin p\theta \cos(p\theta - \alpha) \, d\theta$

Torque experienced by the ampere conductors of the strip $= d\vec{F} \times r$

$d\vec{T} = \hat{A}\hat{B} r l \sin p\theta \cos(p\theta - \alpha) d\theta$

where $r \rightarrow$ radial distance of the conductors from the axis of the shaft.

Torque experienced by the ampere conductors/pole $= \int_{\theta=0}^{\theta=\pi/p} dT$

$$T = \int_0^{\pi/p} \hat{A}\hat{B} r l \sin p\theta \cos(p\theta - \alpha) d\theta$$

$$= \frac{\hat{A}\hat{B} r l}{2} \int_0^{\pi/p} [\sin(2p\theta - \alpha) + \sin\alpha] \, d\theta$$

$$= \frac{\hat{A}\hat{B} r l}{2} \left[-\frac{\cos(2p\theta - \alpha)}{2p} + \theta \sin\alpha \right]_0^{\pi/p}$$

$$= \frac{\hat{A}\hat{B} r l}{2} \left[-\frac{\cos\alpha}{2p} + \frac{\cos\alpha}{2p} + \frac{\pi}{p} \sin\alpha \right] = \frac{\hat{A}\hat{B} r l}{2} \frac{\pi}{p} \sin\alpha$$

The total torque experienced by all armature conductors,

$$T = 2p \times \text{Torque/pole}$$

$$= 2p \cdot \frac{\pi}{p} \frac{\hat{A}\hat{B}rl}{2} \sin\alpha$$

$$T = \pi \hat{A}\hat{B} rl \sin\alpha$$

Torque experienced by the rotor $= -\pi \hat{A}\hat{B} rl \sin\alpha$

$= \pi \hat{A}\hat{B} rl \sin\beta$ where $\beta = -\alpha$

$\beta \to$ power angle or torque angle

Electromagnetic torque developed in a practical BLPM sine wave motor is

$$T = \pi \hat{A}\hat{B} rl \sin\beta$$

$$= \pi \left[\frac{3\sqrt{2}}{\pi} I_{ph} K_{w1} T_{ph}\right] \hat{B} rl \sin\beta$$

$$= 3\sqrt{2} K_{w1} T_{ph} \hat{B} rl \, I_{ph} \sin\beta$$

$$T = 3 \frac{E_{ph}}{\omega_m} I_{ph} \sin\beta$$

where $E_{ph} = \sqrt{2} \hat{B} rl T_{ph} \omega_m$

K_{w1} = winding factor $= K_{s1} K_{p1} K_{d1}$

K_{s1} = skewing factor $= \frac{\sin \sigma/2}{\sigma/2}$ where $\sigma \to$ skew angle (rad)

K_{p1} = pitch factor $= \cos\frac{\alpha}{2}$ where $\alpha \to$ angle of short pitch

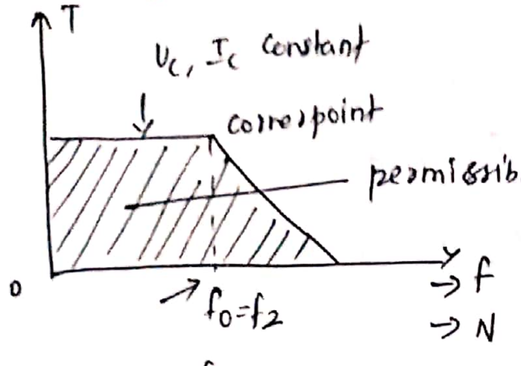
K_{d1} = distribution factor $= \frac{\sin m\beta/2}{m \sin\beta/2}$ where β = slot angle $= 180/n$

n = slots/pole

m = slots/pole/ph.

Ex: Explain the Torque-speed characteristics of permanent magnet synchronous motor.

Soln: ⑤ i)



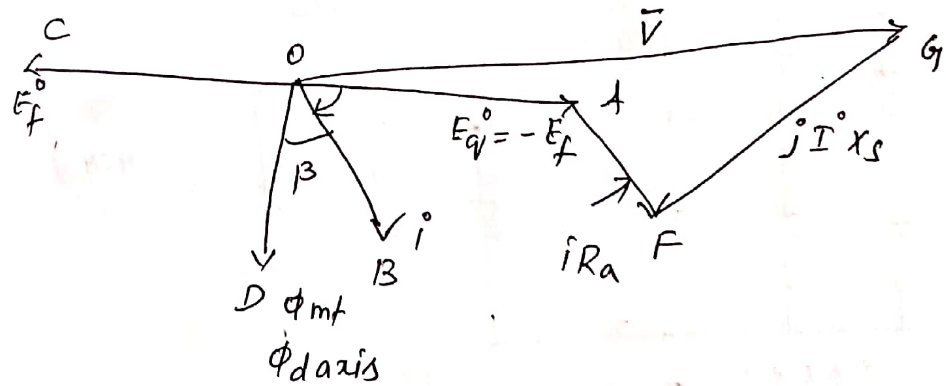
corner frequency $N_0 = \frac{120 f_0}{P}$

For a given maximum permissible voltage (V_c) and maximum permissible current (I_c) the maximum torque remains constant from a low frequency to corner frequency (f_0). Any further increase in frequency decreases the maximum torque. The shaded portion in the torque speed characteristics represents the permissible region of operation.

Ex: Draw the phasor diagram of PMSM and explain it.

Soln:-

⑤ ii)



Let ϕ_{mf} be the mutual flux set up by the permanent magnet, but linked by the armature winding.

E_f^0 lags behind $\phi_{mf} = \phi_d$ axis

AF represents $I_a^0 R_a$, FG represents $I_a^0 X_s$. It is $\perp^r$ to I phasor

OG represents V^0

Angle between the I^0 and ϕ_{mf} is β the torque or power angle power input $= 3\,\dot{V}\cdot\dot{I} = 3\,[\,\dot{E}_q + \dot{I}R_a + j\dot{I}X_s\,]\cdot\dot{I}$ (i) 2

$= \underbrace{3\,\dot{E}_q\cdot\dot{I}}_{\text{EM power}} + \underbrace{3I^2R_a}_{(\text{cul os})}$

EM power transformed as mechanical power

The mechanical power developed $= 3\,\dot{E}_q\,\dot{I}$

$= 3\,|E_q|\,|I|\cos(90-\beta)$

$= 3\,E_q\,I\sin\beta = 3\,|E_f|\,I\sin\beta$

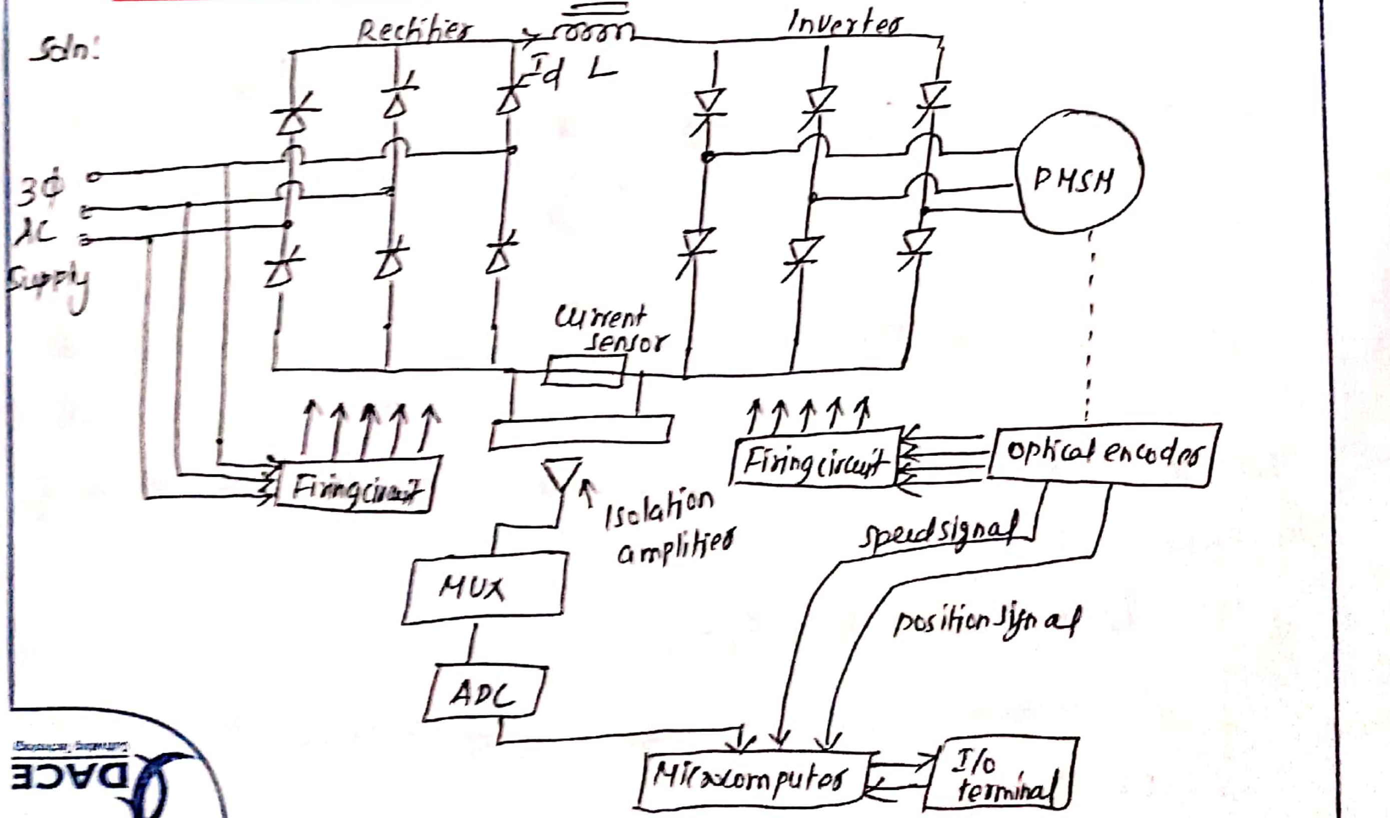
The motor operates at N_s rpm or $\frac{120f}{2P}$ rpm

Electromagnetic Torque developed $= \frac{60}{2\pi N_s} \times 3\,E_q\,I\sin\beta = \frac{P}{\omega m}$

$= \frac{3\,E_q\,I\sin\beta}{\omega_m}$

6) Ex: Explain the microprocessor based control of PMSM with a neat block diagram. (ii) 2

Soln:



To meet the requirements of high demands on control accuracies, flexibility and ease of operation, the microprocessor based control can be used.

The PM synchronous motor is fed from a current source dc link converter system. The system consists of 3ϕ inverter - dc link - 3ϕ rectifier in which the rectifier is fed from a 3ϕ ac supply.

The inverter directs current through the stator phase windings in a controlled sequence. The phase current is sinusoidal function of RPS and an absolute encoder is used to obtain position information with the required resolution.

The programmable counter which is used for sensing the speed is fed with train of pulses having frequency proportional to the motor speed.

The microcomputer gets the system variable signals like rotor position, speed, dc link current etc and the signal from input-output terminal then the control signals to the rectifier and inverter used to get the desired motor performance.

Motor operation is made self synchronous by the addition of a RPS that controls the firing signals for the solid state inverter. The self control of increasing attention, it ensures that the armature and the rotor fields move in synchronism for all operating points. The characteristics of the drive depends on the dc link current, field current, and the inverter firing angle.

The inverter triggering pulses are synchronized to the rotor position signals with a delay angle determined by an 8 bit control input. During normal operation the inverter SCR's are naturally commutated by the machine voltages.

(4) Comparison of PM excitation and electromagnet excitation.

(1)

Permanent magnet Excitation	Electromagnetic excitation
1. Permanent magnets are used in rotor circuit which replaces brushes and slip rings.	There is no use of permanent magnet. The motor has a dc field winding which is supplied from a dc source through slip rings and brushes
2. No field windings are needed so no field current and no continuous field loss	Field losses occur.
3. No field cu losses. This increases efficiency of the machine.	efficiency of the machine is comparatively lower.
4. PM cannot produce as high a flux density as an externally supplied shunt field	Higher flux density can be produced.
5. No possibility of field control	possible of field control
6. Running cost is lower for the machines having PM excitation.	Considering the raw-material cost and present kwh tariffs the running cost is higher.
7. Motor size becomes smaller, as no space is required for field windings.	Larger size
8. require constant power characteristics flux weakening capability is not possible	field weakening is possible.
9. lower induced torque/amp of armature ct	Higher induced torque per ampere of armature current
10. In railway or transit car traction during fault conditions there is a risk of demagnetization of poles which may be caused by large armature currents and overheating.	no such problem arises as windings can be de-excited under fault conditions

(7) ii) State the applications of PM synchronous motor.

(i) Low Integral-hp industrial drives

(ii) fibre spinning mills

(iii) Applied as direct drive traction motor

Used as high speed and high power drives for compressors, blowers, conveyors, fans, pumps, steel rolling mills and aircraft test facilities.