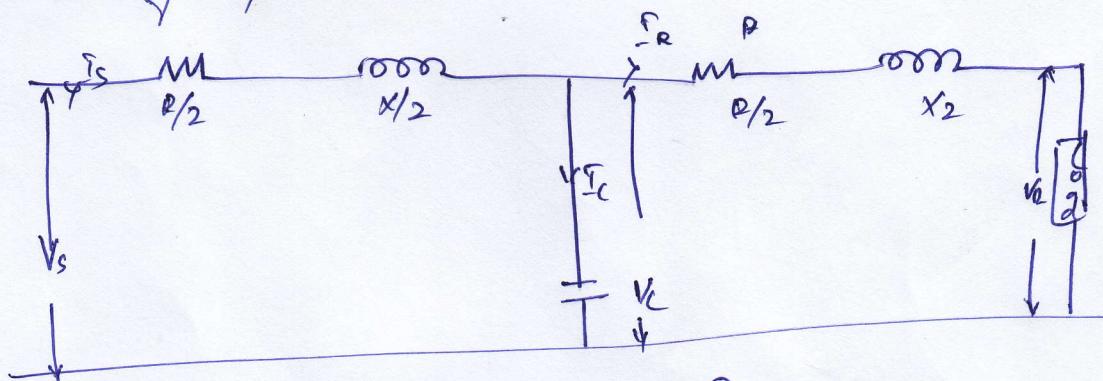


Draw nominal T circuit of a medium length transmission line and derive the expressions for sending end voltage and current also draw the phasor diagram.

In nominal T method the capacitance of each conductor is assumed to be concentrated at the middle of the transmission line and half of the resistance (R) and reactance (X) are assumed to be lumped at either sides of capacitance.



$$\vec{I}_s = \vec{I}_r + \vec{I}_c \rightarrow ①$$

Receiving end voltage as reference.

$$\vec{V}_r = V_r(1+j\alpha)$$

$$\vec{I}_r = \vec{I}_r (\cos \phi_r - j \sin \phi_r)$$

$\cos \phi_r$ = Receiving end power factor.

The voltage across the capacitor V_c is given by

$$V_c = I_r (R/2 + j X/2)$$

$$V_c = I_r + \frac{I_r}{2} (R + j X)$$

The current through the capacitor is given by

$$\vec{I}_c = j \omega C V_c$$

Substitute the value of I_c and I_r in equation (1)

$$\vec{I}_s = \vec{I}_r / (\cos \phi_r - j \sin \phi_r) + j \omega C V_c \rightarrow ②$$

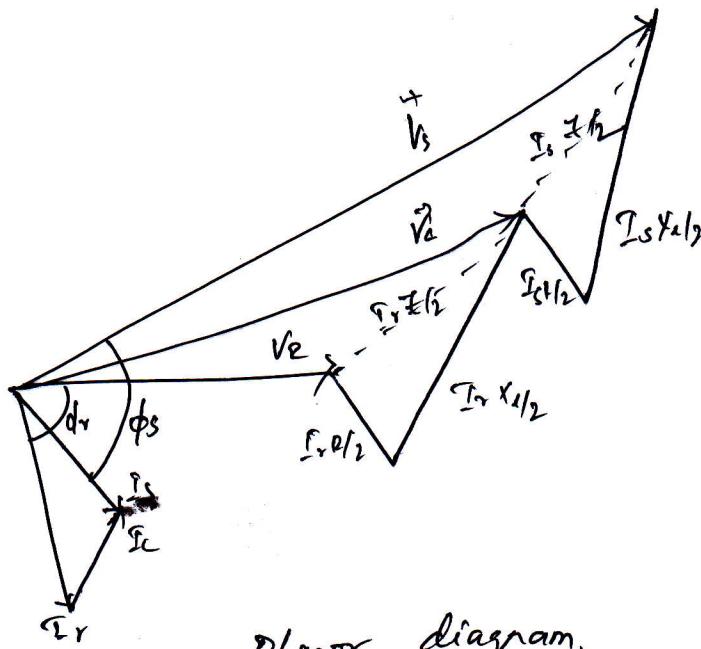
The sending end voltage is given by

$$\vec{V}_s = \vec{V}_r + \frac{\vec{I}_s}{2} (R+jX_2)$$

$$= \vec{V}_r + \frac{\vec{I}_s}{2} (R+jX_1) + \frac{\vec{I}_s}{2} (R+jX_2)$$

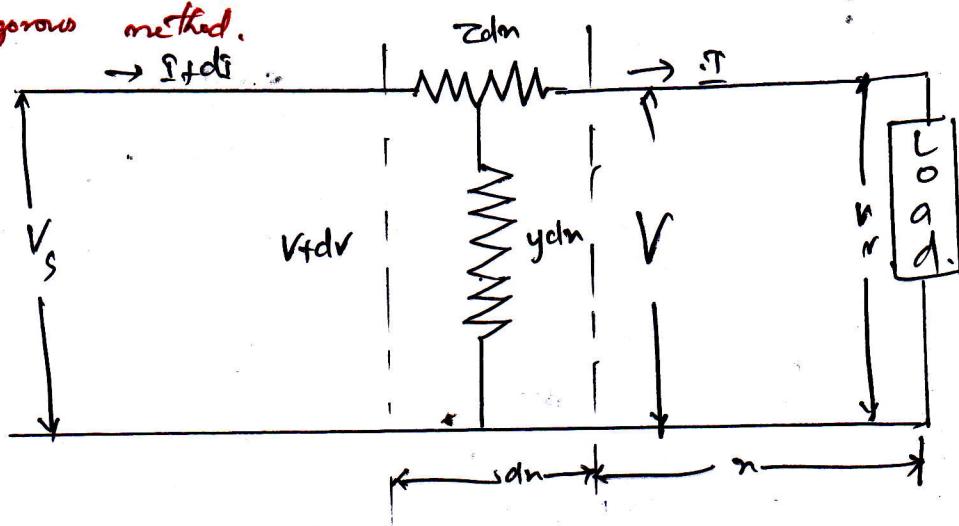
$$= \vec{V}_r + \frac{\vec{I}_s}{2} (R+jX_1) + \frac{1}{2} \left[I_s (\cos \phi_r - j \sin \phi_r) \right] + j \omega C V_r (R+jX_2)$$

Receiving end voltage is taken as reference.



Phasor diagram.

Derive the expression for voltage and current at any point 'x' from the receiving end of a long tr. line. (use rigorous method).



Z = Series impedance per unit length

Y = Shunt admittance per unit length

n = Distance of element dn from receiving end.

V = Voltage at distance n from the receiving end.

V_{ndn} = voltage at a distance ndn from the receiving end.

I = Current leaving the element dn

$\Sigma I dn$ = Current entering the element dn .

V_s, I_s = Sending end Voltage and current

V_r, I_r = Receiving end Voltage and current.

Consider a section of long transmission line dn situated at a distance n from the receiving.

Series impedance of the element dn = $Z dn$.

Shunt input admittance of the element dn = $Y dn$

$$dv = Iz dn.$$

$$\frac{dv}{dn} = \frac{Iz}{Y} \rightarrow \textcircled{1}.$$

$$\frac{dI}{dn} = Vy dn.$$

$$\frac{dI}{dn} = Vy \rightarrow \textcircled{2}.$$

Differentiate equation $\textcircled{1}$ w.r.t. n .

$$\frac{d^2V}{dn^2} = \frac{Z}{Y} \frac{dv}{dn} \rightarrow \textcircled{3}$$

Substitute eqn $\textcircled{2}$ in $\textcircled{3}$

$$\frac{d^2V}{dn^2} = \frac{Z}{Y} I \rightarrow \textcircled{4}$$

Differentiate eqn $\textcircled{2}$ w.r.t. n .

$$\frac{d^2I}{dn^2} = \frac{Y}{Z} V \rightarrow \textcircled{5}$$

Equation ⑦ and ⑥ are mathematically same.
then solution is

$$V = k_1 \cosh(\lambda \sqrt{g} z) + k_2 \sinh(\lambda \sqrt{g} z) \rightarrow ⑦.$$

Differentiate egn ⑦ w.r.t. z.

$$\frac{dv}{dz} = k_1 \sqrt{\frac{g}{z}} \sinh(\lambda \sqrt{g} z) + k_2 \sqrt{\frac{g}{z}} \cosh(\lambda \sqrt{g} z) \quad \text{---} ⑧.$$

$$\frac{du}{dz} = I_z.$$

$$I_z = k_1 \sqrt{\frac{g}{z}} \sinh(\lambda \sqrt{g} z) + k_2 \sqrt{\frac{g}{z}} \cosh(\lambda \sqrt{g} z)$$

$$I = k_1 \sqrt{\frac{g}{z}} \sinh(\lambda \sqrt{g} z) + k_2 \sqrt{\frac{g}{z}} \cosh(\lambda \sqrt{g} z) \rightarrow ⑧.$$

In equation ⑦ and ⑧ initial conditions are used.

At $z=0$ $V=V_r$ and $I=I_r$

$$V_r = [k_1 \cosh(0) + k_2 \sinh(0)] = k_1 + 0.$$

$$I_r = \left[\sqrt{\frac{g}{z}} [k_1 \sinh(0) + k_2 \cosh(0)] \right] = \sqrt{\frac{g}{z}} k_2$$

From above equations.

$$k_1 = V_r \text{ and } k_2 = \sqrt{\frac{g}{z}} I_r$$

substituting the values of k_1 and k_2 in equation ⑦ and ⑧ we get.

$$V = V_r \cosh(\lambda \sqrt{g} z) + \sqrt{\frac{g}{z}} I_r \sinh(\lambda \sqrt{g} z)$$

$$I = \sqrt{\frac{g}{z}} V_r \sinh(\lambda \sqrt{g} z) + I_r \cosh(\lambda \sqrt{g} z)$$

Sending end voltage (V_s) and current (I_s) are obtained by keeping $z=l$.

$$V_s = V_r \cosh(\lambda \sqrt{g} l) + \sqrt{\frac{g}{z}} I_r \sinh(\lambda \sqrt{g} l)$$

$$I_s = \sqrt{\frac{g}{z}} V_r \sinh(\lambda \sqrt{g} l) + I_r \cosh(\lambda \sqrt{g} l)$$

$$\sqrt{Y/Z} = \sqrt{(a_1)(a_2)} = \sqrt{YZ}$$

$$\sqrt{Y/Z} = \sqrt{\frac{Y}{Z}} = \sqrt{YZ}.$$

Y = Total shunt admittance.

Z = Total series impedance.

$$V_s = V_r \cosh(\sqrt{YZ}) + I_r \sqrt{\frac{Z}{Y}} \sinh(\sqrt{YZ})$$

$$I_s = V_r \sqrt{\frac{Y}{Z}} \sinh(\sqrt{YZ}) + I_r \cosh(\sqrt{YZ})$$

$$\vec{V}_s = \vec{A} \vec{V}_r + \vec{B} \vec{I}_r$$

$$\vec{I}_s = \vec{C} \vec{V}_r + \vec{D} \vec{I}_r$$

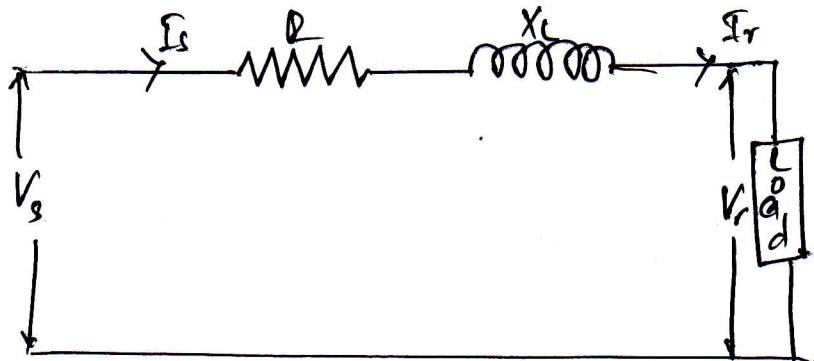
$$A = D = \cosh h \sqrt{YZ}$$

$$B = \sqrt{\frac{Z}{Y}} \sinh h \sqrt{YZ}$$

$$C = \sqrt{Y/Z} \sinh h \sqrt{YZ}$$

3. Draw the phasor diagram of a short transmission line and derive an expression for voltage regulation and transmission efficiency.

For short transmission line the effect of capacitance is negligible because of low charging current arriving to the short length.



V_s and I_s sending end voltage and current
 V_r and I_r Receiving end voltage and current

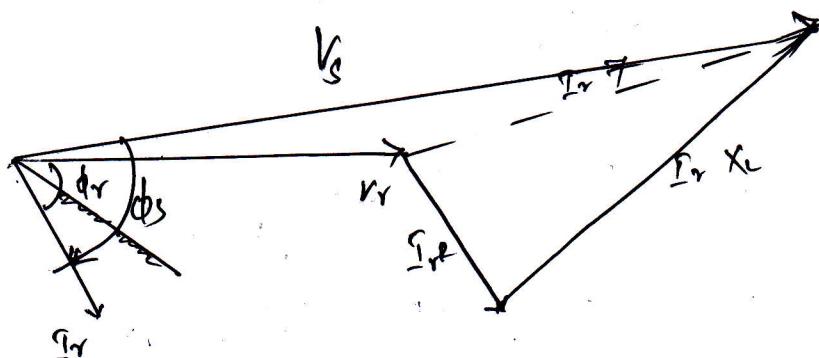
R be the resistance/phane of the line

X be the reactance/phane of the line

ϕ_r be the phase angle between V_r and I_r

ϕ_s be the phase angle between V_s and I_s .

$$\boxed{I_s = I_r}$$



V_R is taken as reference.

$$\vec{V}_S = \vec{V}_R + \vec{E}(R+jX_L).$$

$$\vec{I}_R = \vec{I}_S$$

$$\vec{V}_R = V_R + j0$$

$$\vec{I}_n = I_n (\cos\phi_n - j\sin\phi_n)$$

$$\vec{E} = (R+jX_L)$$

$$\vec{V}_S = \vec{V}_R + \vec{I}_R (\cos\phi_n - j\sin\phi_n) (R+jX_L)$$

$$= \vec{V}_R + (I_R \cos\phi_n - jI_R \sin\phi_n)(R+jX_L)$$

$$= \vec{V}_R + I_R R \cos\phi_n - jR I_R \sin\phi_n + jI_R X_L \cos\phi_n + I_R X_L \sin\phi_n$$

$$\vec{V}_S = \vec{V}_R + \vec{I}_R (R \cos\phi_n + X_L \sin\phi_n) + jI_R (X_L \cos\phi_n - R \sin\phi_n)$$

Transmission efficiency

$$\text{Power delivered} = P_e = V_e I_e \cos\phi_e.$$

Input power = O/P power + losses.

$$= V_e I_e \cos\phi_e + \vec{I}^2 R$$

$$\boxed{X_L = \frac{V_e I_e \cos\phi_e}{V_e I_e \cos\phi_e + \vec{I}^2 R} \times 100.}$$

$$\boxed{\% \text{ Regulation} = \frac{V_S - V_R}{V_R} \times 100.}$$