01. Test of significance of the difference Between Sample Proportion and Population Proportion:

The test statistic $z = \dfrac{p - P}{\sqrt{\frac{PQ}{n}}}$

02. Test of Significance of the difference between two Sample Proportions:

The test statistic $z = \dfrac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$

$Q = 1 - P$

03 Test of Significance of the difference between Sample mean and Population mean:

$z = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$; $H_0: \bar{x} = \mu$; $H_1: \bar{x} \neq \mu$; $H_1: \bar{x} > \mu$; $H_1: \bar{x} < \mu$

04 Test of Significance of the difference means of two Samples:

$z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$ Here $H_0: \bar{x}_1 = \bar{x}_2$; $H_1: \bar{x}_1 \neq \bar{x}_2$; $H_1: \bar{x}_1 > \bar{x}_2$, $H_1: \bar{x}_1 < \bar{x}_2$.

NOTE:-

01 The table value of z

5% level : Two tailed test :- 1.96

one tailed test :- 1.645

02 Type II Error :- Accept Ho when it is False

Type I Error :- Reject Ho when it is True.

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PROBLEMS:-

01. The fatality rate of typhoid patients is believed to be 17.26%. In a certain year 640 patients suffering from typhoid were treated in a Metropolitan hospital and 63 patients died. Can you consider the hospital efficient

Solution:-

$H_0: p = P$ i.e. The hospital is not efficient.

$H_1: p < P$

Given: $P = 17.26\%$ $= 0.1726$; $Q = 1 - P = 1 - 0.1726 = 0.8274$; $p = \frac{63}{640} = 0.0984$

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.0984 - 0.1726}{\sqrt{\frac{0.1726 \times 0.8274}{640}}} = -4.96$$

$\therefore |z| = 4.96$

The table value of z at 5% level (one tailed test) = 1.645

The calculated value of z is greater than the table value. Hence H_0 is Rejected i.e. The Hospital is efficient.

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02. In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant?

Solution:

Given: $P_1 = 20\% = 0.2$; $P_2 = 18.5\% = 0.185$, $n_1 = 900$, $n_2 = 1600$

$H_0: P_1 = P_2$; $H_1: P_1 \neq P_2$

Now, $P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2} = \frac{180 + 296}{900 + 1600} = 0.1904$

$$z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$z = \frac{0.2 - 0.185}{\sqrt{0.1904 \times 0.8096 \times \left(\frac{1}{900}+\frac{1}{1600}\right)}} = 0.92$$

The table value of z at 5% level $= 1.96$

Since the calculated value of z is with in the table value Hence H_0 is accepted. i.e. The difference between P_1 and P_2 is not significant.

03. A Sample of 100 Students is taken from a large Population. The mean height of the Students in this Sample is 160 cm. Can it be reasonably regarded that, in the Population, the mean height is 165 cm, and the SD is 10 cm?

Solution:-

Given, $\bar{x} = 160$, $n = 100$, $\mu = 165$, $\sigma = 10$

$H_0: \bar{x} = \mu$; $H_1: \bar{x} \neq \mu$

$$z = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{160 - 165}{10/\sqrt{100}} = -5$$

$$|z| = 5$$

The table value of z at 5% level is $= 1.96$

Since the calculated value of z is greater than the table value.

Hence H_0 is accepted

i.e. It is not Statistically Correct to assume that $\mu = 165$.

04 A simple sample of heights of 6400 English men has a mean of 170cm and an standard deviation of 6.4 cm. While a simple sample of heights of 1600 Americans has a mean of 172 cm and an SD 6.3 cm. Do the data indicate the Americans are, on the average, taller than the Englishmen?

Solution:

Given $n_1 = 6400$, $\bar{x}_1 = 170$, $S_1 = 6.4$, $n_2 = 1600$, $\bar{x}_2 = 172$, $S_2 = 6.3$

$H_0: \bar{x}_1 = \bar{x}_2$; $H_0: \bar{x}_1 < \bar{x}_2$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{170 - 172}{\sqrt{\frac{(6.4)^2}{6400} + \frac{(6.3)^2}{1600}}} = -11.32$$

$\therefore |Z| = 11.32$ The Table value of Z at 5% level = 1.645.

Since the calculated value of Z is greater than the table value. Hence H_0 is Rejected. (ie) Americans are on the average, taller than Englishmen.

01. Students t - test :-

$$|t| = \frac{\bar{x} - \mu}{S/\sqrt{n}} \quad \text{where} \quad S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Here $H_0: \bar{x} = \mu$; $H_1: \bar{x} \neq \mu$, $H_1: \bar{x} > \mu$, $H_1: \bar{x} < \mu$.

The total no of degrees of freedom $v = n-1$

Note:- If Standard deviation is given $|t| = \frac{\bar{x} - \mu}{S/\sqrt{n-1}}$

02. Student's t test for difference means:-

$$|t| = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \text{where} \quad S^2 = \frac{\sum (x_i - \bar{x}_1)^2 + \sum (x_i - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

Here $H_0: \bar{x}_1 = \bar{x}_2$; $H_1: \bar{x}_1 \neq \bar{x}_2$; $H_1: \bar{x}_1 > \bar{x}_2$, $H_1: \bar{x}_1 < \bar{x}_2$

The total no of degrees of freedom $v = n_1 + n_2 - 2$

Note:- If Standard deviation is given, $S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}$

03. F Test:- [Variance Test] :-

$$F = \frac{S_1^2}{S_2^2} \text{ (or) } \frac{S_2^2}{S_1^2} \text{ where } \quad S_1^2 = \frac{\sum (x_i - \bar{x}_1)^2}{n_1 - 1}$$

$$S_2^2 = \frac{\sum (x_i - \bar{x}_2)^2}{n_2 - 1}$$

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

(or)

$F = \frac{\text{larger variance}}{\text{Smaller variance}}$

The total no of degrees of freedom $v = (n_1 - 1, n_2 - 1)$ (or) $v = (n_2 - 1, n_1 - 1)$

01. Tests made on the breaking strength of 10 pieces of metal wire gave the results 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577 kg.

Solution:

Let $H_0: \bar{x} = \mu\ (577)$; $H_1: \bar{x} \neq \mu$

Now, $\bar{x} = \dfrac{578 + 572 + 570 + 568 + 572 + 570 + 570 + 572 + 596 + 584}{10}$

$= \dfrac{5752}{10} = 575.2$

x	$(x_i - \bar{x})^2$
578	7.84
572	10.24
570	27.04
568	51.84
572	10.24
570	27.04
570	27.04
572	10.24
596	432.64
584	77.44
	681.6

Now, $s^2 = \dfrac{\sum (x_i - \bar{x})^2}{n-1}$

$= \dfrac{681.6}{10-1}$

$s^2 = 75.73$

$s = \sqrt{75.73}$

$= 8.702$

Now, $|t| = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

$= \dfrac{575.2 - 577}{8.702/\sqrt{10}}$

$|t| = \dfrac{-1.8}{2.751}$

$t = 0.654$

The total no. of degrees of freedom $v = n-1$
$= 10-1 = 9$

For 9 degrees of freedom table value of t at 5% level is = 2.26

Since the calculated value is with in the table value. Hence H_0 is accepted ... strength of the wire can be assumed as 577 kg.

02. A certain injection administered to each of 12 patients resulted in the following increases of blood pressure. 5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4

Can it be concluded that the injection will be, in general accompanied by an increase in B.P

Solution:

Let $H_0: \bar{x} = \mu\ (0)$; $H_1: \bar{x} > \mu$

Now, $\bar{x} = \dfrac{5+2+8-1+3+0+6-2+1+5+0+4}{12} = \dfrac{31}{12} = 2.58$

x	$(x-\bar{x})^2$
5	5.8564
2	0.3364
8	29.3764
−1	12.8164
3	0.1764
0	6.6564
6	11.6964
−2	20.9764
1	2.4964
5	5.8564
0	6.6564
4	2.0164
	104.9168

Now, $s^2 = \dfrac{\sum (x_i-\bar{x})^2}{n-1}$

$= \dfrac{104.9168}{12-1}$

$s^2 = 9.537$

$s = 3.088$

$|t| = \dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{2.58-0}{3.088/\sqrt{12}}$

$= \dfrac{2.58}{0.8914}$

$= 2.89$

The total no. of degrees of fredom $v = n-1$
$= 12-1 = 11$

for 11 degrees of freedom the table value for t at 5% level is (one-tailed test)
$= 1.80$

Calculated value of t is greater than table value. Hence H_0 is Rejected.

(ie) we may conclude that the injection is accompained by an increase in B.P

03. A machinist is expected to make engine parts with axle diameter of 1.75 cm. A Random Sample of 10 parts shows a mean diameter 1.85 cm with Standard deviation of 0.1 cm. on the basis of this sample, would you say that the work of the machinist is inferior?

Solution:

Given $\bar{x} = 1.85$, $s = 0.1$, $n = 10$, $\mu = 1.75$

Here $H_0: \bar{x} = \mu$; $H_1: \bar{x} \neq \mu$

$$|t| = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{1.85 - 1.75}{0.1/\sqrt{10-1}} = \frac{0.1}{\frac{0.1}{3}} = 3$$

From the t-table for $\nu = 9$; $t_{0.05} = 2.26$

Since the calculated value of t is greater than the table, Hence H_0 is rejected. Hence the work of the machinist can be assumed to be inferior.

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04. The nicotine contents in two random samples of tobacco are given below.

Sample 01 : 21 24 25 26 27

Sample 02 : 22 27 28 30 31 36

Can you say that the two samples came from the same Population

Solution:

(i) F - Test:-

Here $H_0: \sigma_1^2 = \sigma_2^2$; $H_1: \sigma_1^2 \neq \sigma_2^2$

Now; $$\bar{x}_1 = \frac{21+24+25+26+27}{5}$$

$$= \frac{123}{5} = 24.6$$

$$\bar{x}_2 = \frac{22+27+28+30+31+36}{6}$$

$$= \frac{174}{6} = 29$$

x_1	$(x_1-\bar{x}_1)^2$ $(x_1-24.6)^2$	x_2	$(x_2-\bar{x}_2)^2 = (x_2-29)^2$
21	12.96	22	49
24	0.36	27	04
25	0.16	28	01
26	1.96	30	01
27	5.76	31	04
		36	49
	21.2		108

Now,
$$s_1^2 = \frac{\sum (x_1-\bar{x}_1)^2}{n_1-1} = \frac{21.2}{5-1} = 5.3$$

$$s_2^2 = \frac{\sum (x_1-\bar{x}_2)^2}{n_2-1} = \frac{108}{6-1} = 21.6$$

$$F = \frac{\text{Larger variance}}{\text{Smaller variance}} = \frac{s_2^2}{s_1^2}$$

$$= \frac{21.6}{5.3}$$

$$F_c = 4.07$$

The table value of $F = (n_2-1, n_1-1)$ (ie) $(5, 4)$ degrees of freedom at 5% level is 6.26.

Since the calculated value of F is with in the table value. Hence H_0 is accepted

The variances of the two populations can be regarded as equal

(ii) Students t test:- $H_0: \bar{x}_1 = \bar{x}_2$; $H_1: \bar{x}_1 \neq \bar{x}_2$

Now, $|t| = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Now; $S^2 = \dfrac{\Sigma (x_i - \bar{x}_1)^2 + \Sigma (x_i - \bar{x}_2)^2}{n_1 + n_2 - 2}$

$= \dfrac{21.2 + 108}{5 + 6 - 2}$

$S^2 = 14.35$

$S = \sqrt{14.35} = 3.78$

$|t| = \dfrac{24.6 - 29}{3.78\sqrt{\frac{1}{5} + \frac{1}{6}}}$

$|t| = \dfrac{-4.4}{3.78 \times 0.605}$

$|t| = -1.92$

$|t| = 1.92$

The total no. of degrees of freedom $v = n_1 + n_2 - 2$

$= 5 + 6 - 2$

$= 9$

for 9 degrees of freedom the table value of t at 5% level is $= 2.26$

Since the calculated value of t is not greater than the table value.

hence H_0 is accepted.

(iii) The means of two samples do not differ significantly.

Conclusion:- The two samples could have been drawn from the same normal population.

16. Samples of two types of electric bulbs were tested for length of life and the following data were obtained.

	Size	mean	Standard deviation
Sample 01	8	1234 hours	36 hours
Sample 02	7	1036 hours	40 hours

Is the difference in the means sufficient to warrant type I bulbs are superior to type II bulbs?

Solution:

Given $H_0: \bar{x}_1 = \bar{x}_2$; $H_1: \bar{x}_1 > \bar{x}_2$

and $\bar{x}_1 = 1234$, $S_1 = 36$, $n_1 = 8$, $\bar{x}_2 = 1036$, $n_2 = 7$, $S_2 = 40$.

Now $S^2 = \dfrac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2} = \dfrac{8(36)^2 + 7(40)^2}{8+7-2} = \dfrac{21568}{13} = 1659.07$

$S = \sqrt{1659.07} = 40.73$

$$|t| = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{1234 - 1036}{40.73\sqrt{\frac{1}{8} + \frac{1}{7}}} = \frac{198}{40.73 \times 0.5175} = 9.39$$

The total no. of degrees of freedom $\nu = n_1 + n_2 - 2$

$= 8 + 7 - 2$

$= 13$

For 13 degrees of freedom the table value of t at 5% level is (one-tailed test) $= 1.77$

Since the calculated value of t is greater than the table value

Hence H_0 is rejected.

(a) Type I bulbs may be regarded superior to type II bulbs.

PROBLEMS FOR PRACTICE

01. The following table gives the Biological values of Protein from cow's milk and buffalo's milk at a certain level. Examine if the average values of Protein in the two samples significantly differ.

Cow's milk :	1.82	2.02	1.88	1.61	1.81	1.54	
Buffalo's milk :-	2.00	1.83	1.86	2.03	2.19	1.88	

[Ans:- $\bar{x}_1 = 1.78$, $\bar{x}_2 = 1.965$, $|t| = 2.03$, table value at $v = 10 = 2.23$]

02. Two independent samples of eight and seven items respectively had the following values of the variable.

Sample 01 :-	9	11	13	11	15	9	12	14
Sample 02 :-	10	12	10	14	9	8	10	

Do the two estimates of population variance differ significantly at 5% level of significance?

[$F = \frac{4.79}{3.96} = 1.21$, F (table $(7, 6)$) $= 4.21$. H_0 is accepted]

Chi square test :- χ^2 test :-

$\chi^2 = \sum \frac{(O-E)^2}{E}$, O - observed frequency , E - Expected frequency .

The total no of degrees of freedom $V = n-1$

(or) $V = (r-1)(c-1)$

Uses of χ^2 distribution :-

(i) χ^2 distribution is used to test the goodness of fit.

(ii) It is used to test the independence of attributes.

Problems:

Q1. The following data give the number of aircraft accidents that occured during the various days of a week.

Day :-	Mon	Tues	Wed	Thu	Fri	Sat
No of accidents :-	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week.

Solution:

H_0 : Accidents occur uniformly over the week.

H_1 : Accidents do not occur uniformly over the week

$$E = \frac{15+19+13+12+16+15}{6} = 15.$$

O :	15	19	13	12	16	15
E :	15	15	15	15	15	15
$(O-E)^2$:	0	16	4	9	1	0

$$\therefore \chi^2 = \sum \frac{(O-E)^2}{E} = \frac{0}{15} + \frac{16}{15} + \frac{4}{15} + \frac{9}{15} + \frac{1}{15} + \frac{0}{15} = \frac{30}{15} = 2.$$

The total no of degrees of freedom $V = n-1 = 6-1 = 5$

For 5 degrees of freedom the table value of χ^2 at 5% level = 11.07

Since the calculated of value of χ^2 is with in table value hence H_0 is accepted

02 Theory predicts that the proportion of beans in 4 groups A, B, C, D should be 9 : 3 : 3 : 1 . In an experiment among 1600 beans , the numbers in the 4 groups were 882, 313, 287 and 118. Does the experiment support the theory?

Solution :-

H_0: The experiment support the theory.

H_1: The experiment do not support the theory.

O :	882	313	287	118
E :	$\frac{9}{16} \times 1600 = 900$	$\frac{3}{16} \times 1600 = 300$	$\frac{3}{16} \times 1600 = 300$	$\frac{1}{16} \times 1600 = 100$
$(O-E)^2$:	324	169	169	324

$$\chi^2 = \Sigma \frac{(O-E)^2}{E} = \frac{324}{900} + \frac{169}{300} + \frac{169}{300} + \frac{324}{100} = 4.72$$

The total no. of degrees of freedom $v = n-1 = 4-1 = 3$

For 3 degrees of freedom the table value at 5% level is = 7.82

Since The calculated value is within the table value, hence H_0 is accepted.

i.e. The experimental data support the theory.

03 The following data is collected on two characters. Based on this can you say that there is no relation between smoking and literacy?

	Smokers	Non Smokers
Literates	83	57
Illiterates	45	68

Solution:-

H_0 :- Literacy and smoking habit are Independent.

H_1 : Literacy and Smoking habit are dependent.

Given	Smokers	Non Smokers	Row total
Literates	83	57	140
Illiterates	45	68	113
Column Total	128	125	Grand Total = 253

O	E = Row total x Column total / Grand total		$\frac{(O-E)^2}{E}$	
83	$\frac{140 \times 128}{253} = 70.83$	$\cong 71$	$12^2/71$	$= 2.03$
57	$\frac{140 \times 125}{253} = 69.17$	$\cong 69$	$12^2/69$	$= 2.09$
45	$\frac{113 \times 128}{253} = 57.17$	$\cong 57$	$12^2/57$	$= 2.53$
68	$\frac{113 \times 125}{253} = 55.83$	$\cong 56$	$12^2/56$	$= 2.57$

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 2.03 + 2.09 + 2.53 + 2.57$$

$$= 9.22$$

Total no. of degrees of freedom $\nu = (r-1)(c-1)$

$= (2-1)(2-1) = 1$

For 1 degrees of freedom the table value of χ^2 at 5% level is 3.84

Since the calculated value of χ^2 is greater than the table value

hence H_0 is Rejected

i.e There is some association between literacy and Smoking