UNIT-4

GEARS & GEAR TRAINS

1.) The pressure angle of two gears is 20° & has a module of 10mm. The no of teeth on pinion is 24 and is on gear 60. The addendum of pinion and gear is same equal to one module.

Find:- 1) no of pairs of teeth in contact
2) The angle turned through by the pinion and the gear wheel when one pair of teeth is in contact
3) The ratio of sliding to rolling motion when the tip of a tooth on the large wheel 1) just making contact 2) just leaving contact with its mating tooth 3) at a pitch point [AU/ NOD - 2010]

Given data: $\phi = 20°$, $m = 10mm$, $T_P = 24$, $T_G = 60$, $a_p = a_w = 1 = 10mm$

Sol.

$r = \frac{mT_P}{2} = \frac{10 \times 24}{2} = 120mm$

$R = \frac{mT_G}{2} = \frac{10 \times 60}{2} = 300mm$

$r_A = r + a_p = 120 + 10 = 130mm$

$R_A = R + a_w = 300 + 10 = 310mm$

Length of path of approach

$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{310^2 - 300^2 \cdot \cos^2 20} - 300\sin 20$

$= 26.34mm$

Length of path of recess

$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$= \sqrt{130^2 - 120^2\cos^2 20} - 120\sin 20$

$= 23.64mm$

Length of path of contact

$KL = KP + PL$

$= 26.34 + 23.64$

$= 49.98mm.$

Length of arc of contact

$= \frac{KL}{\cos\phi} = \frac{49.98}{\cos 20}$

$= 53.19mm$

no of pair of teeth in contact

Contact ratio $= \frac{\text{Length of arc of cont}}{\pi m}$

$= \frac{53.19}{\pi \times 10} = 1.69$ pairs

2) Angle turned by the pinion & gear wheel when one pair of teeth is contact

Angle through by pinion = $\frac{\text{length of one of contact}}{\text{circumference of pinion}} \times 360^\circ$

$= \frac{53.19}{2\pi \times 120} \times 360 = 25.39^\circ$

Angle turned by gear wheel.

$= \frac{53.19}{2\pi \times 300} \times 360$

$= 10.16^\circ$

3) Ratio to sliding to rolling motion

Gear ratio $\frac{\omega_P}{\omega_G} = \frac{T_G}{T_P}$

$\omega_G = \omega_P \times \frac{T_P}{T_G}$

$= \omega_P \times \frac{24}{60} = 0.4\,\omega_P$ mm/s

Rolling velocity $V_r = \omega_P \cdot r$

$= \omega_G \cdot R = \omega_P \times 120$

$= 120\,\omega_P$ mm/s

ratio of sliding to rolling velocity.

$\frac{V_{SK}}{V_{rK}} = \frac{36.87\,\omega_P}{120\,\omega_P} = 0.307$

At the pitch point of the length of path of contact is zero. So the sliding velocity will be zero. Therefore the ratio of sliding to rolling velocity is zero.

2) Two mating involute spur gears of 20° pressure angle have a gear ratio of 2. The number of teeth on pinion is 20 & its speed is 250 revolutions per minute. The module pitch of the teeth is 12 mm.

If the addendum of each wheel is such that path of approach & path of recess on each side are half the maximum possible length each side are half the maximum possible length.

1) find addendum for pinion & gear wheel.

2) the length of arc of contact

3) max velocity of sliding during approach & recess

Assume pinion to be driver. [N/D. 2006]

Given data: $T_p = 20$, $G = 2$, $N_p = 250$ rpm, $m = 12$ mm.

We know that, angular velocity of pinion

$$\omega_p = \frac{2\pi N_p}{60} = \frac{2\pi \times 250}{60} = 26.16\ r/s$$

Gear ratio $G = \frac{T_G}{T_P} = 2$

$T_G = 2\,T_P = 2 \times 20 = 40$

$$G = \frac{\omega_P}{\omega_G} = \frac{T_G}{T_P} = 2$$

$$\omega_G = \frac{\omega_P}{2} = \frac{26.16}{2} = 13.08\ r/s$$

Pitch circle radii of pinion & gear wheel.

$$r = \frac{m T_P}{2} = \frac{12 \times 20}{2} = 120\ mm$$

$$R = \frac{m T_G}{2} = \frac{12 \times 40}{2} = 240\ mm$$

1) Addendum for pinion & gear wheel.

$$KP = \frac{1}{2} MP = \frac{r \sin\phi}{2}$$

$$\sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi = \frac{r \sin\phi}{2}$$

$$\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}$$

Substituting the values of R & r in eqn.

$$\sqrt{R_A^2 - 240^2\cos^2 20} - 240\sin 20 = \frac{120\sin 20°}{2}$$

$R_A = 247.77\,mm$

Addendum of gear wheel $a_w = R_A - R$

$= 247.77 - 240 = 7.77\,mm$

Sub. values of R & r

$$\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}$$ 1)

$$\sqrt{r_A^2 - r^2\cos^2 20} - 120\sin 20 = \frac{240\sin 20}{2}$$

$r_A = 139.5\,mm$

Addendum of pinion $a_p = r_A - r$

$= 139.5 - 120 = 19.5\,mm$

2) <u>Length of arc of contact:</u>

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{247.7^2 - 240^2\cos^2 20} - 240\sin 20 = 20.52\,mm$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{139.5^2 - 120^2\cos^2 20} - 120\sin 20 = 41.08\,mm$$

Length of path of contact = KP + PL

$= 20.52 + 41.08 = 61.6\,mm$

Length of arc of contact = ~~20.52~~ $\frac{61.6}{\cos 20} = 65.55\,mm$

③ max velocity of sliding during approach & recess.

$V_{SK} = (\omega_1 + \omega_2) \times$ length of path of approach

$= \omega_1 + \omega_2 \times KP = (26.16 + 13.08) \times 20.52 =$ ~~805.~~

$= 805.2\,mm/s.$

sliding during recess.

$V_{SL} = \omega_1 + \omega_2 \times PL$

$= (26.16 + 13.08)\,41.08 =$ ~~16~~

$= 1611.97\,mm/s.$

3.) Two Gear wheels mesh externally and are to give a velocity ratio of 3. The teeth are of involute form of module 6. The standard addendum is one module. press angle is 20°. and pinion rotates at 90rpm. find.

1) No of teeth on each wheel. interference just avoided.

2) length of path of contact

3) max velocity of sliding b/w teeth.

(M/J-2003, 2006, 2014)

Given:- $\frac{\omega_P}{\omega_G} = \frac{T_G}{T_P} = 3.$, $m = 6mm$, $a_p = a_w = 1$ module

$\phi = 20°$, $N_P = 90$ rpm.

Sol.

$a_w = A_w \cdot m$

$a_w = 1 \times$ module

A_w - addendum co-eff $A_w = A_P = 1$

(1) No of teeth on each wheel.

$$T_G(min) = \frac{2A_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$$= \frac{2 \times 1}{\sqrt{1 + \frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20} - 1} = 44.94$$

No of teeth on pinion

$$T_P = \frac{T_G}{G} = \frac{45}{3} = 15$$

1) length of path of contact.

$$r = \frac{mT_P}{2} = \frac{6 \times 15}{2} = 45.$$

$$R = \frac{mT_G}{2} = \frac{6 \times 45}{2} = 135 \text{ mm}$$

4

Addendum circle radii of pinion & Gear wheel.

$r_A = r + \text{Addendum} = 45+6 = 51\,mm.$

$R_A = R + \text{Addendum} = 135+6 = 141\,mm.$

Length of path of approach:

$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{141^2 - 135^2\cos^2 20^\circ} - 135\sin 20^\circ$

$= 15.37\,mm$

$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$= \sqrt{51^2 - 45^2\cos^2 20} - 45\sin 20$

$= 13.12\,mm$

$KL = KP + PL = 15.37 + 13.12$

$= 28.49\,mm$

(5) max velocity of sliding b/w teeth.

$\omega_P = \frac{2\pi N_P}{60} = \frac{2\pi \times 90}{60} = 9.42\,r/s.$

velocity ratio $= \frac{\omega_P}{\omega_G} = \frac{T_G}{T_P} = 3$

$\omega_G = \frac{\omega_P}{3} = \frac{9.42}{3} = 3.14\,r/s$

we know that max velocity of sliding b/w teeth

$V_s = (\omega_P + \omega_G)\,KP$ as $KP > PL$

$= (9.42 + 3.14) \times 15.37 = 193.05\,mm/s$

1) A Pair of 20° full depth involute spur gear having 30 & 50 teeth respectively. of module 4mm are in mesh. The smaller gear rotates at 1000rpm, Determine.

1) sliding velocities at engagement and at dis engagement of pair of a teeth

2) contact ratio. (AU. N/D: 2004, 2005)

Given: $\phi = 20°$, $T_P = 30$. $T_G = 50$, $m = 4$, $N_P = 1000rpm$

$$G = \frac{T_G}{T_P} = \frac{50}{30} = 1.67$$

Addendum on pinion (a_p)

minimum no of teeth on pinion avoid interference.

$$T_P = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

$$30 = \frac{2A_P}{\sqrt{1 + 1.67(1.67+2)\sin^2 20} - 1}$$

$A_P = 4.655$

Addendum on pinion $a_p = A_P \times m$

$= 4.655 \times 4 = 18.62mm.$

Addendum on gear wheel (a_w)

$$T_G = \frac{2A_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

$$50 = \frac{2A_w}{\sqrt{1 + \frac{1}{1.67}\left(\frac{1}{1.67} + 2\right)\sin^2 20} - 1}$$

$A_w = 2.18$

Addendum on gear wheel

$a_w = A_w \times m$

$= 2.18 \times 4 = 8.72$

$r_A = r + a_p = 60 + 18.62$

$= 78.62 mm.$

$R_A = R + a_w = 100 + 8.72$

$= 108.72 mm$

$KP = \sqrt{R_A^2 - R^2 \cos^2\phi} - R\sin\phi$

$= \sqrt{108.72^2 - 100^2 \cos^2\phi} - r\sin\phi$

$= \sqrt{108.7^2 - 100^2 \cos^2 20} - 100\sin 20$

$= 20.48 mm$

$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$= \sqrt{78.62^2 - 60^2\cos^2 20} - 60\sin 20$

$= 34.27 mm$

$KP + PL = KL$

$= 20.48 + 34.27 = 54.75 mm$

1) Sliding velocity at engagement & disengagement of pair of teeth.

Angular velocity of Pinion

$\omega_P = \frac{2\pi N_P}{60} = \frac{2\pi \times 1000}{60}$

$= 104.72 r/s$

$G = \frac{\omega_P}{\omega_G} = 1.67$

Angular velocity of Gear wheel.

$\omega_G = \frac{\omega_P}{G} = \frac{104.72}{1.75}$

$= 62.71 r/s$

Sliding velocity at the point of engagement K.

$V_{SK} = (\omega_P + \omega_G) KP$

$= (104.72 + 62.71) \times 20.48$

$= 3428.96 mm/s.$

$V_{SL} = (\omega_P + \omega_G) PL$

$= (104.72 + 62.71) 34.27$

$= 5737.83 mm/s$

Contact ratio:

Length of Contact

$= \frac{KL}{\cos\phi}$

$= \frac{54.75}{\cos 20} = 58.26 mm$

Contact ratio = $\frac{\text{Length of path of contact}}{\pi \times m}$

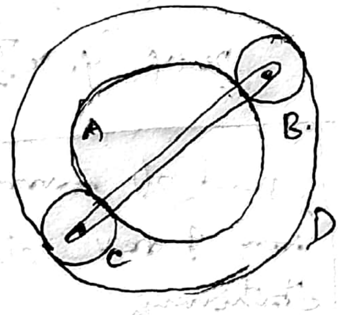
$= \frac{58.26}{\pi \times 4} = 4.64$ pairs.

5.) An epicyclic gear train is arranged as shown in fig. The internal gear D has 90 teeth & the sun gear A has 40 teeth. The two planet gears B & C are identical, and they are attached to an arm as shown.

How many revol does the arm make:

1) when 'A' makes one revolution clockwise & 'D' makes half rev anticlockwise.

2) when 'A' makes one rev clockwise and 'D' remains stationary. (M/J. 2007, A/M. 2010, N/D. 2011)



Given.

$T_A = 40, T_D = 90$

Sol.

$d_D = d_A + d_B + d_C$

$d_D = d_A + 2d_B$

Similarly,

$T_D = T_A + 2T_B$

$90 = 40 + 2T_B$

$2T_B = 90 - 40 = 50$

$\boxed{T_B = 25}$

motion of elements.

Step No.	Operations	Arm	Sun A	Planets B & C	Internal D
		Revolutions of elements			
1.	Fix arm A (+1) rev ACW	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B} \times \frac{T_B}{T_D} = -\frac{T_A}{T_D}$
2.	× x	0	x	$-x\frac{T_A}{T_B}$	$-x\frac{T_A}{T_D}$
3	+y	+y	+y	+y	+y
4	Total	y	x+y	$y-x\frac{T_A}{T_B}$	$y-x\frac{T_A}{T_D}$

1) Gear A 1 rev clock

$x+y=-1$

2) Gear D half Anticlock.

$y-x\frac{T_A}{T_D}=\frac{1}{2}$

$y-x\frac{40}{90}=45$

$90y-40x=45$

$x=-1.038,\ y=0.038$

Speed of arm $y=0.038$

Speed of arm when A 1 rev clock D is stationary.

$x+y=-1$

2) Gear D stationary

$y-x\frac{T_A}{T_D}=0$

$y-x\frac{40}{90}=0$

$90y-40x=0$

$x=-0.6992$ rpm

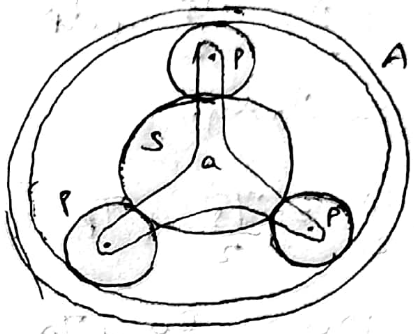
$y=-0.308$ rpm.

Scanned by CamScanner

6) The annulus gear A is shown in Fig. rotates at 300rpm about the axis of fixed wheel S which has 80 teeth. The three-armed spider (only arm a is shown) is driven at 180 rpm. Determine no of teeth required on wheel P. (N/D - 2010)(2017)

Given: $N_A = 300$ rpm, $T_S = 80$,

$N_a = 180$ rpm.



motion of elements.

Step No	Operations	Spider a	Sun S	planet P	Annular A
1.	fix arm S(+1) rev (ACW)	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_P} \times \frac{T_P}{T_A} = -\frac{T_S}{T_A}$
2.	× x	0	x	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_A}$
3.	add +y	y	y	y	y
4	Total	y	x+y	$y - x\frac{T_S}{T_P}$	$y - x\frac{T_S}{T_A}$

Revolutions of elements.

Conditions

1) Spider a rotates 180 rpm

$y = 180$

2) Sun S fixed

$x + y = 0$

3) A rotates 300rpm.

$y - x\frac{T_S}{T_A} = 0$

$x = -4$, $y =$ [illegible]

Sol:-

$80 - (-40) - \frac{80}{\tau_A} = 0$

$\tau_A = 120$

$\frac{\sigma_e}{2} = \frac{\sigma_s}{2} + \sigma_t$

Similarly

$\frac{\tau_A}{2} = \frac{\tau_B}{2} + \tau_P$

$\frac{120}{2} = \frac{80}{2} + \tau_P$

$\tau_P = 20$