

Q1 Explain different types of geometric modeling with suitable examples.

Three types

1. wireframe modeling.
2. Surface modeling.
3. Solid modeling.

1. wire frame modeling :- It's used to generate line model is defined by its lines and their end point coordinates. It's having wide range of application. It is used in low end design systems and manufacturing sys. where its difficult to model complex geometrics by using this method.

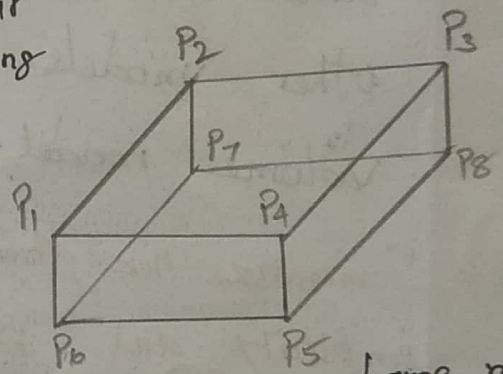


Fig:- wire frame model

2. surface modeling :-

It consists of planes, ruled surfaces and complex surfaces. An object can be effectively represented. when considered with respect to manufacturing. and it is unable to retrieve information of the internal surface. Also it is difficult to compute different properties of the solid model.

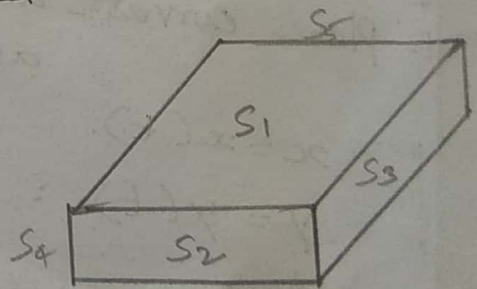


Fig. Surface model.

Bezier and B-splines are techniques used to control and solve the surface models.

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3. Solid modeling:- It is the most extensively used solid representation method.

It provides the complete information about the solid model being represented.

It is easy to interpret data in case of solid model. Compared to other models, solid model is also called as volume model.

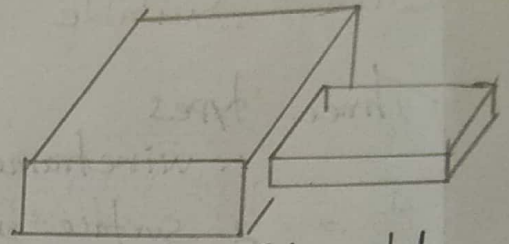


Fig :- solid model.

Q2 Summarize the three representation forms (Parametric, implicit and Explicit) for plane curves, space curves and surfaces. Compare the three representations and write any three inferences.

i. Plane curves - are represented in Parametric form as.

$$x = x(t).$$

$$y = y(t) : t_1 < t < t_2.$$

where

$x(t), y(t)$ - Components of the curve in respective coordinates.

t - parameter.

ii) Representation of plane curves in implicit form is as follows.

$$\text{i.e. } f(x, y) = 0.$$

where x and y are variable.

iii) Representation of plane curves in explicit form as.

$$\text{i.e. } y = f(x)$$

where x and y are two variables and y is the function of x .

2. Space Curves.

i) " are represented in parametric

$$\text{i.e., } \begin{aligned} x &= x(t) \\ y &= y(t) \\ z &= z(t) \end{aligned} \quad \text{where } t_1 \leq t \leq t_2.$$

ii) The space curves are represented in implicitly

$$\text{i.e., } f(x, y, z) = 0 \quad \text{and} \quad g(x, y, z) = 0.$$

iii) The representation of space curves in explicitly

$$y = Y(x) \quad \text{and} \quad z = Z(x).$$

3. Surfaces.

i) The representation of surfaces in parametric

$$\text{i.e. } \begin{aligned} x &= x(u, v) \\ y &= y(u, v) \\ z &= z(u, v) \end{aligned} \quad \text{where } \begin{aligned} u_1 &\leq u \leq u_2 \\ v_1 &\leq v \leq v_2. \end{aligned}$$

ii) The representation of surfaces in implicit.

$$\text{i.e. } f(x, y, z) = 0. \quad \text{where } f(x, y, z) = 0$$

iii) The representation of surfaces in explicitly

$$z = F(x, y)$$

where x, y, z are three variables and z is the function of two variables x and y .

Comparison.

Parametric Form

i) closed and multivalued curves and infinite slopes can be represented.

ii) Since axis is independent, it is easy to transform.

iii) Easy to trace the curve.

iv) Intersections and point classification is difficult due to high flexibility.

v) Composite curves can also be generated easily.

vi) Easy to fit and manipulate free shapes.

Implicit form

closed and multivalued curves and infinite slopes can be represented.

Since axis is dependent, difficult to transform.

Complex to trace the curve.

Point classification is easy, i.e. solid modelling and interference checking.

Difficult to fit and manipulate free forms shapes.

Explicit Form

If $f(x)$ is a polynomial, infinite slopes can not be represented.

Since axis is dependent, difficult to transform.

Easy to trace the curve.

Following three inferences are observed among the comparison.

1. Among three forms of representation, Parametric form is the most versatile method and explicit form is least used method.

2. Explicit form can be easily converted to Parametric form.

3. Implicit form is more complex than Parametric form.

Q. 3. Explain different features of a Bezier curve with construction details.

(or)

What are Bezier curves? Discuss its important properties.

Ans:

Bezier curves are used in geometric modeling are obtained based on either interpolation techniques or approximation techniques. The curves generated by interpolation techniques, pass through given or specified data points, whereas, the curves generated by approximation techniques does not pass through the specified set of data points. Approximation techniques are preferred over interpolation techniques, does not pass through the specified, due to its flexibility. One such example of curves produced by approximation techniques is Bezier curves.

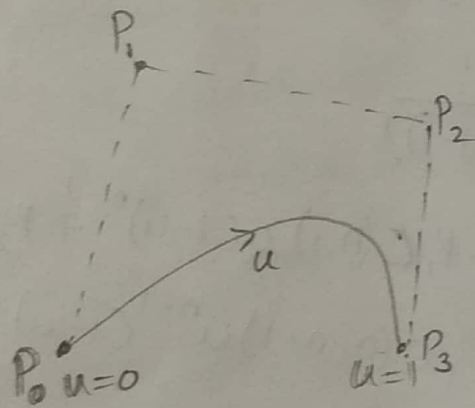
Bezier curves are developed by the founder of french car firm Regie Renault, P. Bezier. These curves are used in UNISURF software, to design the outer panels of the car.

Bezier curves are different from cubic splines in the following aspects.

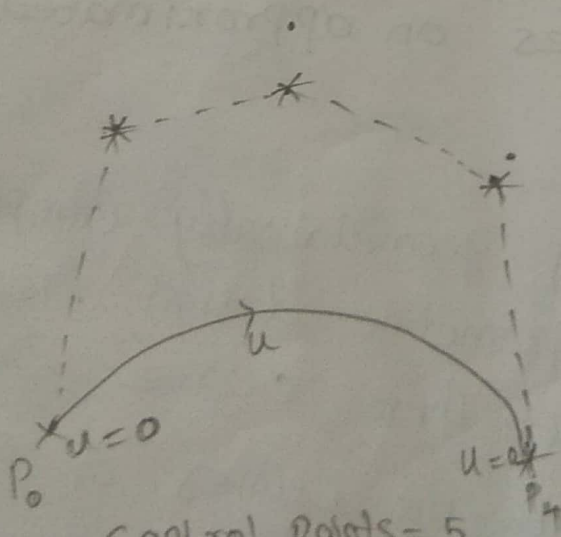
1. Specified data points only can control the shape of bezier curve. In cubic-splines, 1st order derivatives are used to define the curve, while in Bezier curve, they are not used, which enables the designer to understand the relation between data points and curve easily.
2. Specified data points only can control the shape of bezier curve. ~~In cubic~~
3. The degree of Bezier curve varies with the number of data points, whereas the degree of cubic splines is always constant.
4. Bezier curve is more smooth comparatively, due to its higher order derivatives.
5. Bezier curves can be crossed or cross over by changing the order of control points.

For a Bezier curve, $(n+1)$ data point define n^{th} degree curve. And, only first and last data or control points are present on the curve

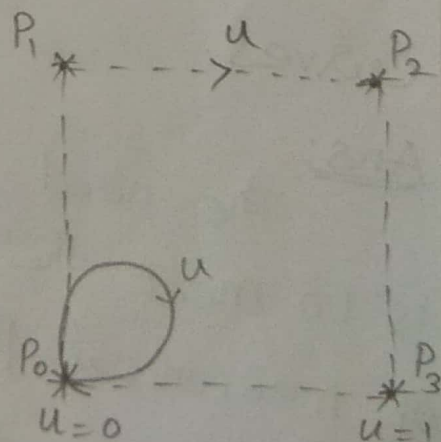
and the curve is tangent to the lines passing through these points. Intermediate data points, define the order, derivative and controls the shape of the curve. The shape of the curve also follows the shape of the polygon. order of defining the control points, determines the shape of the curve, as shown in the following figure. The following figures represent different shapes of Bezier Curve



Control Points = 4,
Degree of curve = 3



Control points = 5
Degree of curve = 4



Control points = 4
Degree of curve = 3

Mathematical eqn of a Bezier curve with $(n+1)$ control points and n^{th} degree is,

$$P(u) = \sum_{i=0}^n P_i B_{i,n}(u); \quad 0 \leq u \leq 1$$

where

$P(u)$ - Any point on the curve

P_i = Control Point

$B_{i,n}$ = Bernstein Polynomials

$$B_{i,n}(u) = C(n,i) u^i (1-u)^{n-i}$$

where

$C(n,i)$ - Binomial Co-efficient and,

$$C(n,i) = \frac{n!}{i!(n-i)!}$$

$$\therefore P(u) = P_0(1-u)^n + P_1 C(n,1) u(1-u)^{n-1} + P_2 C(n,2) u^2(1-u)^{n-2} + \dots + P_{n-1} C(n,n-1) u^{n-1}(1-u) + P_n u^n$$

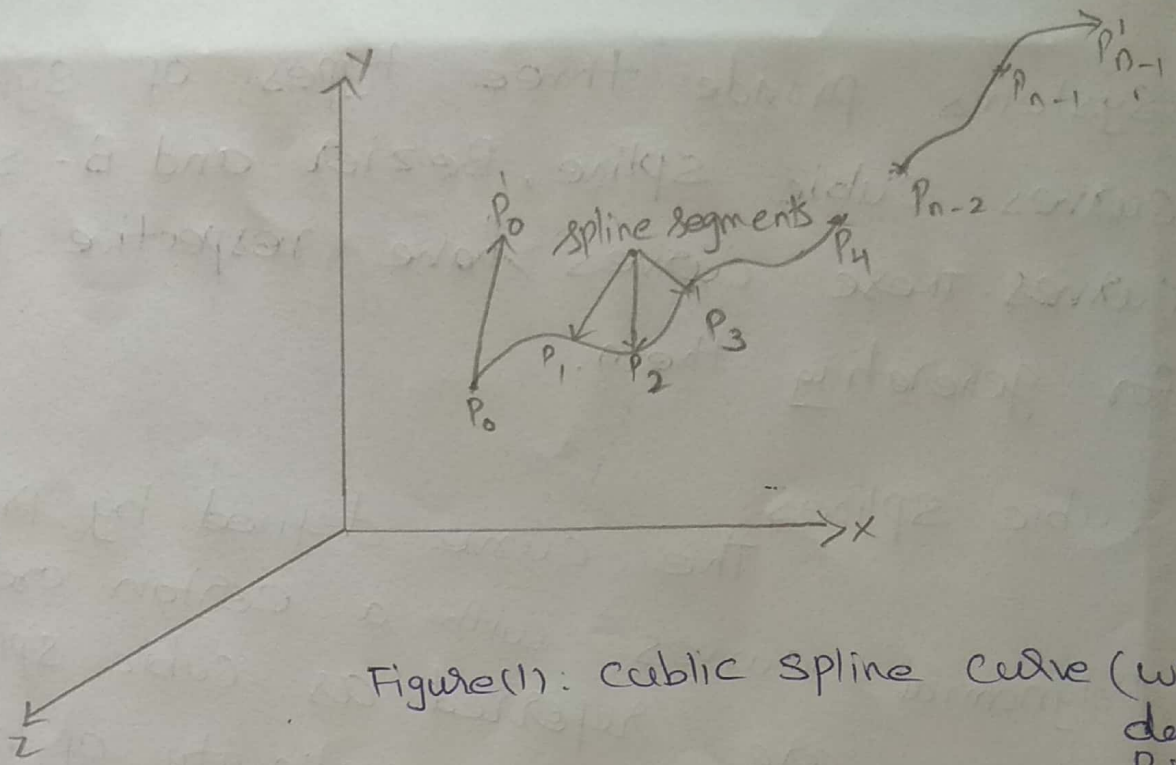
Q4) Write short notes on approximated Synthetic curves.

Ans: The need of geometrically complex curves is to meet the geometric design requirements of mechanical parts. These curve require free-form or synthetic curves or surface to get accurate design. Most of the CAD/CAM

with systems provide three types of synthetic curves, cubic spline, Bezier and B-spline curves. These curves have respective methods for generating them.

1. Cubic Splines

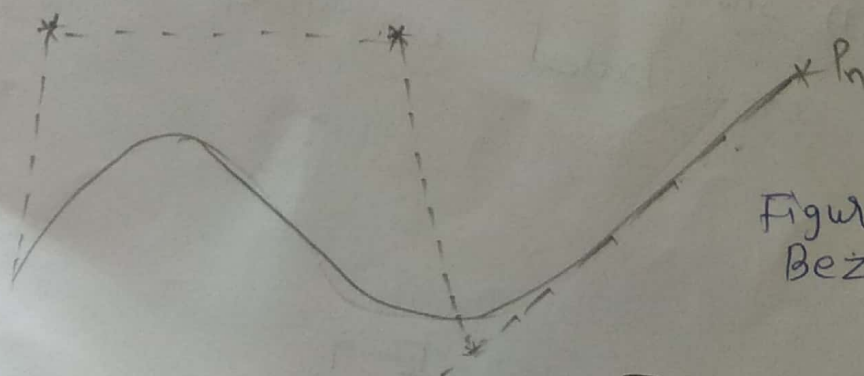
The curve defined by piece-wise Polynomial curves with a certain order of continuity are referred as cubic splines. A Polynomial of degree n has continuity of $(n-1)^{th}$ order derivatives. The cubic spline curve passes through the data points and hence it is an interpolant. This curve is defined by a given set of set data points as shown in figure (1). If there is any change in the position of data points or end slope, it changes the entire shape of the spline curve. These are not much popular. Figure (1) shows a typical cubic spline curve. These are based on interpolation techniques.



Figure(1): Cubic spline curve (with data points)

2. Bezier curve:

These curves are created by using approximation techniques, which does not pass through the given data points. And, the points (data points) are used to control the shape of the curve. These are defined in terms of the location of $(n+1)$ points. These points form the control or Bezier characteristic polygon which defines the curve shape as shown in the figure (2)



Figure(2)
Bezier curve

In partial applications, there may arise the need for complex curve, then various curves may be joined (blended) together to meet this requirement.

3. B-spline curve:

This curve has the ability to interpolate or approximate a set of given data points. A greater degree of flexibility can be obtained by the B-spline curves. This includes both interpolation and approximation techniques to get a design of free-form curve.

i) Approximate a Given set of Data Points

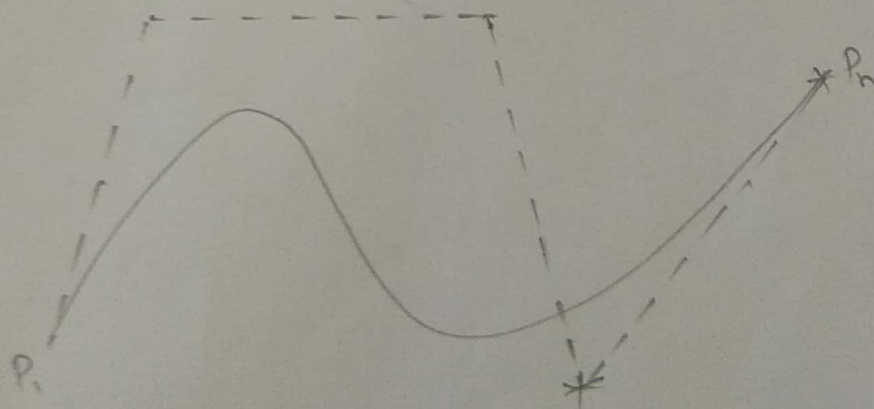


Figure (3)

ii, Interpolate a given set of Data Points

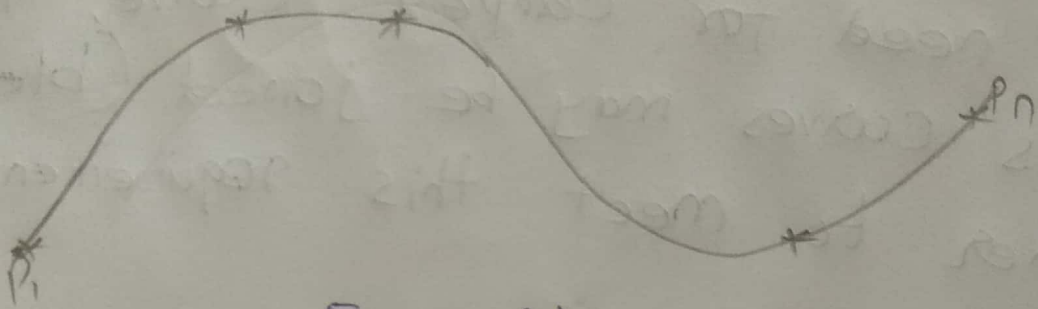


Figure (4)